

DISSERTATION

zum Thema

**Quantum Field Theory Near
Thresholds of Ultra-Heavy Particles:
Top-Antitop and Beyond.**

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Für meine Familie

Kurzfassung

Das Standardmodell der elektroschwachen und starken Wechselwirkung (SM) beschreibt bis heute fast alle experimentellen Daten innerhalb der experimentellen und theoretischen Unsicherheiten. Zu seiner inneren Konsistenz, dh. im speziellen zur Anomaliefreiheit benötigt das SM ein sechstes, zu Beginn dieser Arbeit noch nicht entdecktes Quark, das Topquark (t). In den letzten beiden Jahren (1994-95) wurde dann die Evidenz für seine tatsächliche Existenz durch Experimente am Hadronbeschleuniger Tevatron immer stärker, sodaß heute der Nachweis bei einer Masse von ca. 180 GeV als gesichert gilt.

Der ideale Platz für eine genauere Untersuchung der Eigenschaften des Topquarks wäre aber ein e^+e^- -Beschleuniger. Dort könnte man Top-Antitop Paare ($t\bar{t}$) nahe der Schwelle erzeugen und so seine Masse, Zerfallsbreite und die Kopplungen zu anderen Elementarteilchen genau bestimmen. Durch die hohe Masse und den raschen Zerfall in W^+ und b besitzt das Topquark die einzigartige Eigenschaft unter den Quarks, daß sein Verhalten rein störungstheoretisch bestimmt werden kann. Die störungstheoretische Behandlung der Bindungszustandseffekte nahe der Schwelle für die Erzeugung ist Gegenstand der vorliegenden Arbeit.

Eine der einfachsten Erweiterungen des SM ist das sog. Minimale Supersymmetrische SM (MSSM). Die darin vorhergesagten skalaren Partner zu den existierenden Fermionen könnten in ähnlicher Weise erzeugt werden und werden daher in dieser Arbeit auch behandelt.

Im Kapitel 2 werden zuerst sowohl für instabile Fermionen als auch für Skalare Bindungszustandsgleichungen angegeben, die als Grundlage für eine systematische Störungstheorie dienen. Als Beispiel wird das Massenspektrum zweier gebundener Skalare zu $O(\alpha^4)$ berechnet. Dann wird das $t\bar{t}$ -Potential in konsistenter Weise zu numerischer Ordnung $O(\alpha_s^4)$ bestimmt. Dabei wurden ein neuer Box-graph berechnet sowie Beiträge der elektroschwachen Wechselwirkung berücksichtigt. Im Rest des Kapitels wird dann die Frage einer möglicherweise großen Korrektur zum Wirkungsquerschnitt auf Grund einer laufenden Zerfallsbreite behandelt. Es kann gezeigt werden, daß ein bisher nicht berücksichtigter Beitrag zu großen Kompensationen führt. Durch die Einführung der neuen Gleichung für instabile Fermionen und der Verwendung einer Wardidentität läßt sich schließlich die Rechnung soweit vereinfachen, daß der allgemeine Mechanismus dieser Kompensationen sichtbar wird. Dies führt zu dem Theorem, daß für alle nicht durch

Annihilation zerfallenden schwach gebundenen Systeme die Bindungszustandskorrekturen zur Zerfallsbreite allein durch die Zeitdilatation beschrieben werden könne.

Kapitel 3 beschäftigt sich mit der Berechnung der Wirkungsquerschnitte nahe der Schwelle mit numerischen Methoden. Zuerst wird der allgemeine Formalismus beschrieben und der totale Wirkungsquerschnitt für die $t\bar{t}$ Produktion diskutiert. Dabei zeigt sich unter anderem die Wichtigkeit der großen Zerfallsbreite für eine störungstheoretische Behandlung einerseits sowie die Bedeutung der Resummierung im Gluonpropagator andererseits. Dann wird die Vorwärtsrückwärtsasymmetrie nahe der Schwelle erstmals ohne die Einführung eines künstlichen Cut-offs berechnet. Die dabei verwendete Methode läßt sich auch auf die Berechnung des axialen Beitrags zum totalen Wirkungsquerschnitts beim $t\bar{t}$ -System und die Bestimmung des führenden Beitrags zur Stop-Antistoppproduktion anwenden.

Konnten in dieser Arbeit wichtige Aspekte der Produktion schwerer, stark wechselwirkender Teilchen nahe der Schwelle geklärt werden, so bleiben doch noch einige theoretische Herausforderungen für die Zukunft. Dabei sei speziell die Anwendung der rigorosen Methoden aus Kap.2 auf die Berechnung physikalischer Observablen wie in Kap.3 erwähnt. Dies wird notwendig werden, um Theorie und Experiment mit Genauigkeiten von kleiner als 1% zu vergleichen.

An dieser Stelle möchte ich dem Institut für Theoretische Physik für die freundliche Aufnahme und meinen Freunden und Kollegen für viele wertvolle Diskussionen danken. Ganz besonderer Dank gilt auch meinem Lehrer Herrn O.Univ.Prof. Dr. W.Kummer für seine Unterstützung und viele wertvolle Diskussionen.

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Chapter 1

Introduction

Perturbative expansions in the coupling constant in quantum field theory possess two types of applications, the calculation of scattering amplitudes and the computation of processes involving weakly bound systems. Many of the successes of quantum electrodynamics (QED) are, in fact, related to positronium, i.e. to the second one of the aforementioned applications. The proper starting point for any bound-state calculation in quantum field theory is an integral equation, comprising an infinite sum of Feynman graphs. The Bethe-Salpeter (BS) equation [1] fulfills this task and it is well known that in the limit of binding energies of $O(\alpha^2 m)$ the Schrödinger equation with static Coulomb attraction is obtained. The computation of higher order corrections to the Bohr-levels, however, turned out to be far from trivial. It was recognized, though, relatively late that, at least conceptually, substantial progress with respect to a systematic treatment results from a consistent use of a perturbation theory geared to the original BS equation [2]. As an additional bonus one avoids nonrelativistic expansions as implied by Hamiltonian approaches with successive Foldy-Wouthuysen transformations [3]. Within the BS-technique, however, it is desirable to have an exactly solvable zero order equation different from the Schrödinger equation, because otherwise e.g. the approximation procedure for the wave function lacks sufficient transparency, especially in higher orders and may even break down in certain cases, one of which is treated here. One of the advantages of the BS approach to perturbation theory is the freedom to select a different zero order equation. Of course, in that case, certain corrections included already at the zero level are to be properly subtracted out in higher orders. An especially useful zero order equation has been proposed some time ago by Barbieri and Remiddi (BR equation [4]). Still, one of the

most annoying features of all bound-state calculations remains the pivotal rule played by the Coulomb gauge. In other gauges, e.g. already the (in QED vanishing) corrections to $O(\alpha^3 m)$ of the Bohr levels imply to take into account an infinite set of Feynman graphs [5]. Only in very special cases, when certain subsets of graphs can be shown to represent a gauge-independent correction, another more suitable gauge may be chosen.

By contrast to QED the vast literature on bound state problems in quantum chromodynamics (QCD) adheres to a description of the quark-antiquark system by the Schrödinger equation with corrections 'motivated' by QCD [6]. As long as a relatively small number of parameters suffices for an adequate phenomenological description of observed quantum levels, this approach undoubtedly has an ample practical justification. Furthermore even for the relatively heavy flavors charm and bottom nonperturbative effects can be shown to be of the same order of magnitude as the leading perturbative ones. Apart from lattice calculations there exists no method to do bound state calculations in a non-perturbative way. Only if the non-perturbative contributions are smaller than the leading perturbative ones it is known how to include them in a systematic BS perturbation theory [7, 8]. In the case of QCD this requires particles with masses in the range of 100GeV. Especially the recent evidence for the top quark with a mass around 170 GeV seems to indicate that this particle is the ideal testing ground for perturbative QCD bound state calculations. The cleanest environment to observe the top quark clearly would be a next generation e^+e^- linear collider. On such a machine other heavy particles with strong interaction could be discovered as well. If supersymmetry is realized in nature it seems very likely that one of the scalar partners of either top or bottom could be observed. Since it may well be that even in, say, a 500GeV collider those new scalar particles (and also the top quark) will be produced near their threshold, a reliable prediction of the production cross section slightly above threshold seems very important. The signal of the top quark is expected to be best suited for an exact determination of its mass, its width and an independent measurement of the strong coupling constant.

Thus also for this reason a return to more rigorous QCD arguments remains as desirable as ever. The standard literature on quarkonia (see e.g. [9]) is almost exclusively based on nonrelativistic expansions [3] or on the calculation of purely static forces [10]. Moreover, very often potentials with higher order corrections as determined from on-shell quarkonia scattering are used [11]. In these cases relevant off-shell effects which are typical for higher order corrections may even be lost altogether. On the other hand, from the point of view of relativistic quantum field theory as elaborated in the abelian case of

positronium, a similar, more systematic approach seems desirable, the more so because the basic techniques are well developed. In addition, at least in one case, namely the decay of S-wave quarkonium, the result of a full BS-perturbation calculation [12], including the QCD corrections to the bound state wave function, yields a result very different from the one which took into account only the corrections to the quark antiquark annihilation alone [13].

In this context the relatively large size of the running coupling constant even at high energies represents a well known problem, together with large coefficients from a perturbative expansion. Therefore e.g. problems arise in the comparison of the coupling constant as determined from scattering experiments within the minimal subtraction scheme (\overline{MS}), with the coupling constant to be used in a consistent weak bound-state approach. The philosophy within our present work will be that the orders of magnitude, as determined from $\alpha_{\overline{MS}}$ will be used for estimates, but that we shall imply a determination of α_s by some physical observable of the quarkonium system itself. In that way delicate correlations of 'genuine' orders of α_s from *basically* different types of experiments are avoided.

From high precision electro-weak experiments of the LEP collaborations, the mass range of the top quark now seems to be established to lie in the range $160\text{-}200\text{GeV}$ [14], in agreement with the direct searches at the tevatron [15]. Thus for the first time a nonabelian bound state quarkonium system seems to fulfill the high mass criterion required for a genuine field theoretical approach. Unfortunately the drawback of this situation is that the weak decay $t \rightarrow b + W$ broadens the energy levels [16] for increasing mass m_t so that above $m_t \approx 120\text{GeV}$ individual levels effectively disappear.

But this disadvantage turns into a virtue since it completely eliminates nonperturbative and renormalon contributions. The reason for this can be stated as follows. The time scale Λ_{QCD}^{-1} necessary to allow strong interactions to become effective and to produce e.g. hadrons with open top cannot compete with the decay time Γ_0^{-1} . Therefore the top quark decays before nonperturbative strong interactions can act on it. In this situation "pure" quantum chromodynamic (QCD) perturbation theory is sufficient for a complete description of a strongly interacting system. This applies as well to Coulombic QCD bound-state effects in toponium although also there the hydrogen-like levels are strongly smeared out by Γ_0 . Nevertheless, the computation of those levels represents a necessary first step since the order of magnitude of these corrections can be used as a order parameter for the determination of the quark-antiquark potential. Furthermore we will see in chapter 2 of this work that a lot of insight into the physics of toponium can be gained

from the study of the properties of the four point function at the bound state poles.

After all, this is the first instance where the application of quantum field theory for weakly bound nonabelian systems is justified because even nonperturbative effects can be taken into account in an equally well-defined perturbative manner.

A correct formulation of QCD in Coulomb gauge entails not only Faddeev-Popov-ghost terms but also the inclusion of nonlocal interaction terms [17]. Therefore, the full Lagrangian reads ($a=1,\dots,8$ for SU(3)):

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + \sum_{j=1}^f \bar{\Psi}_j(i\gamma D - m_j)\Psi_j + B^a(\partial_j A_j^a) - \bar{\eta}^a \partial^i (\delta_{ab} \partial_i + g f_{abc} A_i^c) \eta^b + v_1 + v_2 \quad (1.1)$$

where the Lagrange multiplier B^a imposes the Coulomb gauge condition, and where

$$D_\mu = \partial_\mu - ig T^a A_\mu^a, \quad (1.2)$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f_{abc} A_\mu^b A_\nu^c. \quad (1.3)$$

v_1 and v_2 are given in [17] and will be discussed more explicitly below. The above Lagrangian will include all effects of the strong interaction, but, as we will show, QED and weak corrections may also give contributions within the numerical order of our main interest. This effects will be discussed within the framework of the Standard Model (SM). With respect to possible extensions of the SM we will restrict ourselves to the Minimal Supersymmetric SM (MSSM) where we especially focus on the occurrence of new heavy scalars like the stop and the lightest (uncharged) higgs boson.

The thesis is organized as follows. In chapter 2 we first construct a solvable zero order equation for decaying particles (sect. (2.1)-(2.2)). Then in sect. (2.4) we calculate within the framework of Bethe-Salpeter perturbation theory (sect.2.3) the potential for the top-antitop ($t\bar{t}$) system to numerical order $O(\alpha_s^4)$. As a further application of these methods we derive the bound state corrections to the toponium decay width in sect. (2.5).

Chapter 3 is devoted to the calculation of cross sections to be measured in a next generation linear collider. Within the Green-function approach we study some consequences of the results of chapter 2 for the total cross section (sect. 3.1.1). Furthermore in sect. 3.1.2 we propose a method to circumvent an unphysical singularity in the calculation of the forward-backward asymmetry. The same method may also be applied to the calculation of the axial contribution to the $t\bar{t}$ total cross section, as well as to the production of heavy scalar particles near threshold (sect. 3.2).

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Chapter 2

Application of Rigorous Bound State Methods to Toponium

2.1 A Relativistic Equation for Decaying Fermions

In this section we will present a solvable relativistic equation for decaying fermions, similar in form to that for stable fermions of Barbieri and Remiddi [4]. While this equation is applicable to variety of bound state problems we will focus especially on the $t\bar{t}$ system.

The Bethe-Salpeter (BS) approach for weakly bound systems starts from the BS-equation

$$\begin{array}{c}
 \begin{array}{c} i \xrightarrow{p_1} \text{---} \bigcirc \text{---} \xrightarrow{\bar{p}_1} \bar{i} \\ j \xrightarrow{p_2} \text{---} \bigcirc \text{---} \xrightarrow{\bar{p}_2} \bar{j} \end{array} \\
 \text{G}
 \end{array}
 =
 \begin{array}{c}
 \begin{array}{c} i \xrightarrow{p_1} \text{---} \bigcirc \text{---} \xrightarrow{\bar{p}_1} \bar{i} \\ j \xrightarrow{p_2} \text{---} \bigcirc \text{---} \xrightarrow{\bar{p}_2} \bar{j} \end{array} \\
 \text{D}
 \end{array}
 +
 \begin{array}{c}
 \begin{array}{c} i \xrightarrow{p_1} \text{---} \bigcirc \text{---} \bigcirc \text{---} \xrightarrow{\bar{p}_1} \bar{i} \\ j \xrightarrow{p_2} \text{---} \bigcirc \text{---} \bigcirc \text{---} \xrightarrow{\bar{p}_2} \bar{j} \end{array} \\
 \text{D} \quad \text{K} \quad \text{G}
 \end{array}
 \quad (2.1)$$

for the two fermion Green function $G(P, p, p')$, where D is the product of the two (full) propagators and K represents the 2pi BS-kernel. All four point functions in 2.1 depend on the total momentum $P = (P_0, \vec{0})$; the incoming (outgoing) lines carry momentum $P/2 \pm p$

$(P/2 \pm p')$.

Near a bound state pole the Green function assumes the form

$$G_{ij,\bar{i}\bar{j}}(p_1, p_2, \bar{p}_1, \bar{p}_2) = (2\pi)^4 \delta^4(\bar{P} - P) \sum_l \chi_{ij}(p, n, l) \frac{i}{P_0 - M_n} \bar{\chi}_{\bar{i}\bar{j}}(\bar{p}, n, l) + G_{reg} \quad (2.2)$$

where $\chi_{ij}(p, n, l)$ is called BS- or bound state wave function.

Perturbation theory starts from an equation similar to eq. 2.1 with G_0, D_0 and K_0 chosen in such a way that the exact solution is known. If D_0 consists of the nonrelativistic propagators only and if K_0 is the Coulomb kernel the zero order equation corresponding to 2.1 (after integrating out p_0, p'_0 etc.) simply reduces to the Schrödinger equation. A very convenient zero order equation which already includes the relativistic free fermion propagators and still remains solvable has been found by Barbieri and Remiddi (BR) some time ago [4]. However if the decay width of the constituents becomes comparable to the binding energy of $O(\alpha^2 m)$, perturbation theory runs into troubles because of the occurrence of terms $\Gamma/(\alpha^2 m)$ in graphs like fig. 2.1, containing a chain of subgraphs which are responsible for the decay. To circumvent these difficulties one has to include at least a part of the exact self energy function in the zero order equation.

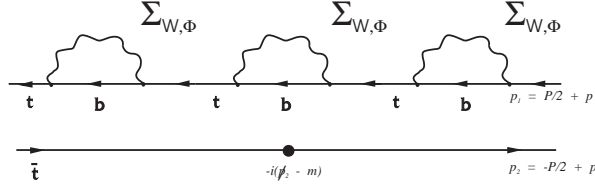


Fig. 2.1

The BS equation in terms of Feynman amplitudes for K and S reads for a bound state wave function χ

$$\chi_{ij}^{BS}(p; P) = -i S_{ii'}(\frac{P}{2} + p) S_{j'j}(-\frac{P}{2} + p) \int \frac{d^4 p'}{(2\pi)^4} K_{i'j', i''j''}(P, p, p') \chi_{i''j''}^{BS}(p'; P), \quad (2.3)$$

where S is the exact fermion propagator, and K is the sum of all two fermion irreducible graphs. Furthermore, we have introduced relative momenta p and p' , a total momentum $P = p_1 - p_2$, and we choose the center of mass (CM) frame where $P = (P_0, \vec{0}) = (2m + E, \vec{0})$. The Green function reads in terms of field operators

$$\begin{aligned} G_{ij,\bar{i}\bar{j}}(p_1, p_2, \bar{p}_1, \bar{p}_2) &:= \int d^4 x_1 d^4 \bar{x}_1 d^4 x_2 d^4 \bar{x}_2 e^{i(p_1 x_1 + \bar{p}_2 \bar{x}_2 - p_2 x_2 - \bar{p}_1 \bar{x}_1)} G_{i,j;\bar{i},\bar{j}}(x_1, x_2, \bar{x}_1, \bar{x}_2) \\ G_{ij,\bar{i}\bar{j}}(x_1, x_2, \bar{x}_1, \bar{x}_2) &:= \langle 0 | T \bar{\Psi}_i(x_1) \Psi_j(x_2) \bar{\Psi}_{\bar{i}}(\bar{x}_1) \bar{\Psi}_{\bar{j}}(\bar{x}_2) | 0 \rangle \end{aligned} \quad (2.4)$$

In terms of Feynman amplitudes and the relative coordinates this can be written as

$$\begin{aligned}
G_{ij,\bar{i}\bar{j}}(p_1, p_2, \bar{p}_1, \bar{p}_2) &= i(2\pi)^4 \delta(P - \bar{P}) G_{ij,\bar{i}\bar{j}}(P, p, p') \\
G_{ij,\bar{i}\bar{j}}(P, p, p') &= \int d^4X d^4x d^4\bar{x} e^{i(XP + xp - \bar{x}\bar{p})} \langle 0 | T \bar{\Psi}_i(X + \frac{x}{2}) \Psi_j(X - \frac{x}{2}) \bar{\Psi}_{\bar{i}}(-\frac{\bar{x}}{2}) \bar{\Psi}_{\bar{j}}(\frac{\bar{x}}{2}) | 0 \rangle
\end{aligned} \tag{2.5}$$

The notation can be read off the pictorial representation in fig 2.2. i, j are collective indices for spin (σ, ρ) and color (noted α, β).

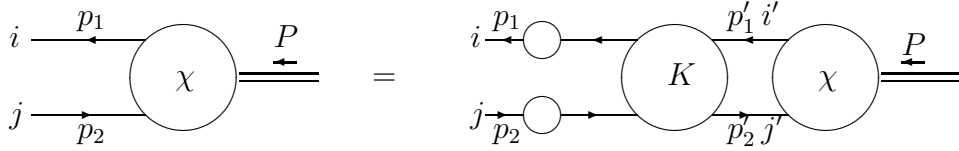


Fig. 2.2

It is well known that the dominant part in K for weak binding ($\alpha \rightarrow 0$) is the one-Coulomb gluon exchange which results in an ordinary Schrödinger equation with static Coulomb potential. With the help of the nonrelativistic scaling argument [18]

$$\begin{aligned}
p_0 &\approx O(m\alpha^2), |\vec{p}| \approx O(m\alpha), \\
P_0 &\approx 2m - O(m\alpha^2)
\end{aligned} \tag{2.6}$$

this result is even independent of the chosen gauge:

$$K \rightarrow K_c \gamma^0 \otimes \gamma^0 := -\frac{4\pi\alpha}{(\vec{p} - \vec{p}')^2} \gamma_{\sigma\sigma'}^0 \gamma_{\rho'\rho}^0 \tag{2.7}$$

Focusing on QCD we notice the fact that only color singlet states can form bound states because the Coulomb potential is repulsive for color octets. The color trace will always be understood to be already done, leading to the definition

$$\alpha \equiv \frac{4}{3} \frac{g^2}{4\pi} = \frac{4}{3} \alpha_s \tag{2.8}$$

to be used in the following, in terms of the usual strong coupling constant α_s . Because the above mentioned nonrelativistic limit of the BR equation contains the projection operators λ^\pm , defined below, it is awkward to calculate the so-called relativistic corrections in a straightforward way within the framework of BS perturbation theory, starting from 2.3

with 2.7. Therefore we will use the generalized BR equation as described below instead of the Schrödinger equation.

Moreover, for our present case, we need a generalization of the equation given in [4] for unstable fermions, described by complex $m \rightarrow \tilde{m} = m - i\hat{\Gamma}_0/2$ where $\hat{\Gamma}_0/2$ represents the imaginary part of the self energy graph responsible for the weak decay of the free top quark. Since the real part of \tilde{m} should still determine the sum of polarizations in the numerator of the propagator, we take by analogy to the stable case in the zero order approximation to 2.3 the relativistic free propagator ($E_p = \sqrt{\vec{p}^2 + m^2}, \eta = m\hat{\Gamma}_0/2E_p$)

$$S(\pm \frac{P_0}{2} + p_0) \rightarrow [(\pm \frac{P_0}{2} + p)\gamma - \tilde{m}]^{-1} = \quad (2.9)$$

$$= \frac{\Lambda^+ \gamma_0}{\pm \frac{P_0}{2} + p_0 - E_p + i\eta} + \frac{\Lambda^- \gamma_0}{\pm \frac{P_0}{2} + p_0 - E_p - i\eta} + O(\frac{\hat{\Gamma}_0}{m}) \quad (2.10)$$

with the relativistic projectors

$$\Lambda^\pm(\vec{p}) \equiv \frac{E_p \pm (\vec{\alpha}\vec{p} + \beta m)}{2E_p}. \quad (2.11)$$

If furthermore $\partial K_0/\partial p_0 = 0$ both sides of 2.3 may be integrated with respect to p_0 . On the r.h.s. the product of the two propagators S with 2.9 yield four terms, with Cauchy poles determined by $i\eta$, two of which give no contribution. Generically we obtain

$$\int \frac{dp_0}{2\pi i} S \otimes S = \frac{\Lambda^+ \gamma_0 \otimes \Lambda^- \gamma_0}{P_0 - 2E_p + 2i\eta} - \frac{\Lambda^- \gamma_0 \otimes \Lambda^+ \gamma_0}{P_0 + 2E_p - 2i\eta} \quad (2.12)$$

so that $K_0(\vec{p}, \vec{p}')\Phi_0$ with

$$\Phi_0 = \frac{1}{2\pi} \int \chi_0 dp_0 \quad (2.13)$$

remains to be inserted at the places of " \otimes ". Written with indices the direct product notation can be explained as: $A \otimes B := A_{\alpha\alpha'} B_{\beta'\beta}$. As in [4] the kernel is now chosen as

$$K_0\Phi_0 = [\gamma_0 \Lambda^+ \lambda^+ \Lambda^+] \Phi_0 [\Lambda'^- \lambda^- \Lambda^- \gamma_0] \tilde{K} \quad (2.14)$$

so as to annihilate the second term in 2.12. In

$$(P_0 - 2E_p + 2i\eta)\Phi_0 = \int \frac{d^3 p'}{(2\pi)^3} \tilde{K}(\vec{p}, \vec{p}', P_0, i\eta) \Lambda^+ \lambda^+ \Lambda^+ \Phi_0 \tilde{\Lambda}'^- \lambda^- \tilde{\Lambda}^- \quad (2.15)$$

instead of Λ^- the projector $\tilde{\Lambda}^- := \Lambda^-(-\vec{p})$ appears. Thus expanding $\Phi_0 = \sum_{A,B=\pm} \Lambda^A \tilde{\Phi}_0^{(AB)} \tilde{\Lambda}^B$ only $\Phi_0^{(+ -)}$ is found to differ from zero and obeys

$$(P_0 - 2E_p + 2i\eta)\Lambda^+\Phi_0^{(+ -)}\tilde{\Lambda}^- = \int \frac{d^3p'}{(2\pi)^3} \tilde{K}(\vec{p}, \vec{p}', P_0, i\eta)\Lambda^+\lambda^+\Phi_0^{(+ -)}(\vec{p}')\lambda^-\tilde{\Lambda}^-. \quad (2.16)$$

The nonrelativistic projection of 2.16 onto $\lambda^+ \otimes \lambda^-$ with

$$\begin{aligned} \lambda^+\Lambda^+\lambda^+ &= \frac{\lambda^+}{\mu} \\ \lambda^-\tilde{\Lambda}^-\lambda^- &= \frac{\lambda^-}{\mu} \\ \mu &= \frac{2E_p}{E_p + m}, \end{aligned} \quad (2.17)$$

and the introduction of appropriate factors in \tilde{K} relatively to the Coulomb kernel 2.7

$$\begin{aligned} \tilde{K} &= \mu\mu'\nu\nu'K_c \\ \nu^2 &= 4/(P_0 + 2E_p + 2i\eta) \end{aligned} \quad (2.18)$$

and in the wave function

$$\Phi_0(\vec{p}) \propto \nu^{-1}\mu^{-1}\phi(\vec{p}) \quad (2.19)$$

also for the case of an unstable fermion lead to a Schrödinger equation for the wave-functions $\Phi(\vec{p})$ in momentum space ($\tilde{E} = P_0 - 2m + 2i\eta = E + 2i\eta$):

$$[\frac{\vec{p}^2}{m} - \hat{E}_n]\phi(\vec{p}) = \frac{\alpha}{(2\pi)^2} \int \frac{d^3p'}{(\vec{p} - \vec{p}')^2} \phi(\vec{p}') \quad (2.20)$$

The eigenvalues for $\hat{E}_n = \tilde{E} + \frac{\tilde{E}^2}{4m}$ in 2.20 clearly occur at the real Bohr levels E_n , i.e.

$$\tilde{E}_n = M_n^0 - 2m = -\frac{m\alpha^2}{4n^2} - \frac{m\alpha^4}{64n^4} + O(\alpha^6); \quad M_n^0 = 2m\sqrt{1 - \sigma_n^2} \quad \sigma_n^2 = \frac{\alpha^2}{4n^2} \quad (2.21)$$

In addition to the selection of the "large" components by the choice 2.14, obviously also the sign of $i\eta$ in 2.18 was crucial for the dependence of 2.20 on the combination $E + 2i\eta$ alone. Still, for the bound-state argument at complex values of the energy, the independence of η with respect to the momentum \vec{p} is essential. Going back to 2.9 we observe that with the choice

$$\hat{\Gamma}_0 = \frac{E_p}{m}\Gamma, \quad (2.22)$$

where $\Gamma = \text{const}$ is the (c.m.) decay rate of a single top quark, the disturbing \vec{p} -dependence in η is cancelled. This leads to

$$\eta = \frac{\Gamma}{2}.$$

The full BS-wave function (color singlet, BS-normalized [19]) for the BR-kernel can be obtained by going backwards to $\chi_0^{(+)} := \chi(p)$. It appears as the real residue of the complex pole. The wave function ($\omega_n = E_p - M_n^0/2 - i\epsilon$)

$$\chi_n(p, \epsilon) = \gamma_0 \bar{\chi}_n^*(p, -\epsilon) \gamma_0 = i \frac{\Lambda^+ S \tilde{\Lambda}^-}{(p_0^2 - \omega_n^2)} \frac{\mu(p)}{\nu(p)} \frac{2\omega_n}{\sqrt{P_0}} \phi(\vec{p}). \quad (2.23)$$

is identical to the BR wave-function of stable quarks and belongs to the spectrum of bound states $P_n = M_n^0 - i\Gamma_t$. The $i\epsilon$ has been introduced to determine the integration around that pole.

In eqs. 2.23 S is a constant 4×4 matrix which represents the spin state of the particle-antiparticle system:

$$S = \begin{cases} \gamma_5 \lambda^- & : \text{ singlet} \\ \vec{a}_m \vec{\gamma} \lambda^- & : \text{ triplet.} \end{cases} \quad (2.24)$$

ϕ is simply the normalized solution of the Schrödinger equation in momentum space, depending on the usual quantum numbers (n, l, m) [20], $a_{\pm 1}, a_0$ in 2.24 describe the triplet states. In the following it will often be sufficient to use the nonrelativistic approximations of eqs. 2.23 ($\omega_n \approx \tilde{\omega}_n = (\vec{p}^2/m + E_n)/2 - i\epsilon$)

$$\chi(p)^{nr} = \frac{\sqrt{2}i\tilde{\omega}_n}{p_0^2 - \tilde{\omega}_n^2} \phi(\vec{p}) S = -\gamma_0 \bar{\chi}(p)^{nr} \gamma_0 \quad (2.25)$$

A similar calculation is needed in order to obtain the solution for the zero order approximation to the BS-equation itself. Written in terms of Feynman amplitudes this equation reads:

$$iG_0 = -D_0 + D_0 K_0 G_0, \quad (2.26)$$

It can be shown that the solution to this equation is

$$\begin{aligned} G_0 &= i(2\pi)^4 \delta(p - p') D_0 + \frac{2\omega}{p_0^2 - \omega^2} [\Lambda^+ \lambda^+ (\Lambda^+)' \gamma_0] \otimes [\gamma_0 \Lambda^- \lambda^- (\Lambda^-)'] \times \\ &\times \mu \mu' \left[\frac{(2\pi)^3 \delta(\vec{p} - \vec{p}')}{2\omega} - \frac{G_C(\hat{E}, \vec{p}, \vec{p}')}{m\nu\nu'} \right] \frac{2\omega'}{(p'_0)^2 - (\omega')^2} \end{aligned} \quad (2.27)$$

where

$$\hat{E} = \frac{(P_0 + i\Gamma)^2 - 4m^2}{4m}, \quad (2.28)$$

$$\omega = E_k - \frac{P_0 + i\Gamma}{2}, \quad (2.29)$$

and G_C denotes the Coulomb Green function, given e.g. in [21].

In the following we will calculate corrections to the position of the bound state poles of this Green function. One may therefore ask whether it is possible to write it as a sum over all resonances and the continuum. The Coulomb Green function G_C clearly can be decomposed in that way, but inserting this spectral representation into 2.27 leads to an expansion with P_0 dependent wave functions

$$G_0 = \sum_n \frac{\chi_n(P_0, p) \bar{\chi}_n(P_0, p')}{P_0^2 - P_n^2} + \tilde{D} \quad (2.30)$$

with

$$\begin{aligned} \tilde{D} = & i(2\pi)^4 \delta(p - p') D_0 + \\ & (2\pi)^3 \delta(\vec{p} - \vec{p}') \frac{2\omega}{(p_0^2 - \omega^2)^2} [\Lambda^+ \lambda^+ (\Lambda^+)' \gamma_0] \otimes [\gamma_0 \Lambda^- \lambda^- (\Lambda^-)'] \mu^2 \end{aligned} \quad (2.31)$$

$$\chi_n(P_0, p) = i \frac{\Lambda^+ S \tilde{\Lambda}^-}{(p_0^2 - \omega^2)} \frac{\mu(p)}{\nu(p)} 2\sqrt{2}\omega \phi_n(\vec{p}). \quad (2.32)$$

The summation over n represents the sum over all quantum numbers: discrete (n, l, m, S, M_S) and continuous (k, l, m, S, M_S) . We emphasize that ω now depends on the total momentum P_0 and not on the fixed energy P_n . Furthermore also the normalization of the wave functions had to be changed due to the different denominator in 2.30 compared to 2.2.

It is interesting to note that the wave functions 2.32, in contrast to the wave functions 2.23, also fulfill an orthogonality relation for different principal quantum numbers.

$$tr \left[\int \frac{d^4 p}{(2\pi)^4} \bar{\chi}_n(P_0, p) \frac{2D_0^{-1}}{P_0^2 - E_p^2} \chi_m(P_0, p) \right] = \delta_{nm} \quad (2.33)$$

The net result of this subsection is that the width of a particle can also consistently be included for a *relativistic* zero order equation similar to the nonrelativistic case where the replacement $E \rightarrow E + i\Gamma$ in the Green function has been proposed first [42]. Here we have the replacement $P_0 \rightarrow P_0 + i\Gamma$ except in the free propagator for the small components.

2.2 An Exactly Solvable Zero Order Equation for Scalar-Scalar Bound States

In the last section we developed the bound state formalism for decaying particles. Until today only fermionic matter fields have been discovered, but in supersymmetric theories for each fermion two scalar partners are required. Since some of them, probably stop or sbottom, could have masses within the reach of the next generation of e^+e^- accelerators, even the observation of bound states of those particles seems possible. These objects and systems built of scalar composite particles in atomic physics underline the need of an equally clear and transparent approach as presented above for the fermionic case. A recent attempt in this direction [22] splits the boson propagator in a particle and anti-particle propagator to be able to treat them like fermions. The spectrum is then obtained by constructing the Hamiltonian via a Foldy - Wouthuysen transformation and a perturbation theory à la Salpeter. This approach suffers from several drawbacks. First it will break down in higher orders due to the appearance of higher powers in \vec{p} and second the introduction of a "new" propagator raises questions on whether one is really calculating consequences of the original field theory. A difference in our result to that of ref. [22] seems to confirm this suspicion.

To the best of our knowledge there exists no attempt in the literature to construct a solvable zero order equation for the BS equation containing two charged scalars interacting via a vector field. Thus we will generalize the approach of the foregoing chapter to this case.

2.2.1 Stable Particles

As starting point we present here an exactly solvable equation for stable scalar particles which interact via a vector field and show how to calculate the fine structure of such a system within our approach.

Starting from eq. 2.3 we would like to use beside the relativistic scalar propagators the kernel due to the Coulomb interaction

$$K_C(p, p') = 4\pi\alpha \frac{(P_0 + p_0 + p'_0)(P_0 - p_0 - p'_0)}{(\vec{p} - \vec{p}')^2}. \quad (2.34)$$

This kernel has the drawback that it is p_0 dependent and the exact solution of eq. 2.3 with 2.34 is not known. However, in the nonrelativistic regime by the scaling argument

(eq. 2.6) we can start with an instantaneous approximation to the kernel since p_0 is of $O(\alpha^2 m)$ in this region and may be included in the corrections afterwards. Then we can perform the zero component integration on the propagator

$$\begin{aligned} & -i \int \frac{dp_0}{2\pi} \frac{1}{[(\frac{P_0}{2} + p_0)^2 - E_p^2 + i\epsilon][(-\frac{P_0}{2} + p_0)^2 - E_p^2 + i\epsilon]} = \\ & = \frac{1}{2E_p P_0} \left[\frac{1}{2E_p - P_0} - \frac{1}{2E_p + P_0} \right] = \frac{1}{E_p(4E_p^2 - P_0^2)} \end{aligned} \quad (2.35)$$

and it is quite easy to show that

$$K_0(p, p') = 4\pi\alpha \frac{4m\sqrt{E_p E_{p'}}}{\vec{q}^2} \quad (2.36)$$

gives a solvable equation with the normalized solutions

$$\chi(p) = i \frac{\sqrt{E_p}(P_0^2 - 4E_p^2)}{\sqrt{2P_0}[(\frac{P_0}{2} + p_0)^2 - E_p^2 + i\epsilon][(-\frac{P_0}{2} + p_0)^2 - E_p^2 + i\epsilon]} \phi(\vec{p}) \quad (2.37)$$

$$\bar{\chi}(p, \epsilon) = -\chi^*(p, -\epsilon) \quad (2.38)$$

to the eigenvalues

$$P_0 = M_n^{(0)} = 2m\sqrt{1 - \sigma_n^2}. \quad (2.39)$$

Eq. 2.38 is dictated by the requirement that $\bar{\chi}$ should acquire the same analytic properties as the underlying field correlators

$$\chi(p) = \int e^{ipx} \langle 0 | T \Phi^\dagger(\frac{x}{2}) \Phi(-\frac{x}{2}) | P_n \rangle, \quad (2.40)$$

$$\bar{\chi}(p) = \int e^{-ipx} \langle P_n | T \Phi(\frac{x}{2}) \Phi^\dagger(-\frac{x}{2}) | P_n \rangle. \quad (2.41)$$

Using the integral representation for the step function which is included in the time ordered product, one derives eq. 2.38.

Taking the equation for the Green function 2.26 instead of that for the BS wave function and using again 2.36 we find

$$G_0 = -F(p) \frac{G_C(\hat{E}, \vec{p}, \vec{p}')}{4m} F(p') \quad (2.42)$$

with

$$\hat{E} = \frac{P_0^2 - 4m^2}{4m} \quad (2.43)$$

and

$$F(p) = \frac{\sqrt{E_p}(P_0^2 - 4E_p^2)}{[(\frac{P_0}{2} + p_0)^2 - E_p^2 + i\epsilon][(-\frac{P_0}{2} + p_0)^2 - E_p^2 + i\epsilon]} \quad (2.44)$$

These solutions can be used for a systematic BS perturbation theory for scalar constituents, as will be demonstrated in the next section.

2.2.2 Unstable Particles

As in the fermionic case it is desirable to construct a bound state equation for decaying particles. This can be done by the replacement

$$E_p \rightarrow \sqrt{E_p^2 - i\Gamma m}. \quad (2.45)$$

While 2.45 leads to expressions for the BS wave functions which contain very unpleasant expressions for the particle poles it has the advantage the propagator has the form as expected from the phase space of a unstable particle. Furthermore the above calculation remains essentially unchanged if we define the square root in 2.45 to be that with the positive imaginary part. Only the energy in the resulting Schrödinger equation and thus in 2.42 changes to

$$\hat{E} = \frac{P_0^2 - 4m^2}{4m} + i\Gamma. \quad (2.46)$$

The eigenvalues for P_0 are

$$P_{0,n} = 2m\sqrt{1 - \sigma_n^2 - i\frac{\Gamma}{m}} \approx M_n^{(0)} - i\Gamma. \quad (2.47)$$

In the case of the fermions we managed to construct wave functions independent of Γ . This was possible because the small components of the propagator containing $P_0 - i\Gamma$ instead of $P_0 + i\Gamma$ were projected to zero with the help of an appropriate kernel. This cannot be achieved in the scalar case and thus, surprisingly enough, the scalar wave functions look more complicated than the fermionic ones. To illustrate this we will present here an approach in close analogy to the fermionic case. Let us consider eq. 2.35. To obtain an expression like eq. 2.12 we define the zero order propagator

$$\begin{aligned} D_0(P_0, p) &= \Delta_+(P_0, p)\Delta_-(P_0, p) \\ \Delta_{\pm}(P_0, p) &= \frac{1}{2E_p} \left[\frac{1}{\pm\frac{P_0}{2} + p_0 - E_p^2 + i\frac{\Gamma}{2}} - \frac{1}{\pm\frac{P_0}{2} + p_0 + E_p^2 - i\frac{\Gamma}{2}} \right]. \end{aligned} \quad (2.48)$$

In order to be able to use the wave functions of the Schrödinger equation with a Coulomb potential we choose a kernel

$$K_0 = \sqrt{\frac{p_+ p'_+}{\nu_s \nu'_s}} \frac{4m\sqrt{\rho\rho'}}{\vec{q}^2 E_p E_{p'}} \quad (2.49)$$

with

$$\rho = E_p - i\frac{\Gamma}{2} \quad (2.50)$$

$$p_{\pm} = 2E_p \pm P_0 - i\Gamma, \quad (2.51)$$

$$\nu_s = 2E_p + P_0 + i\Gamma. \quad (2.52)$$

This leads to the eigenvalue equation

$$\chi(\vec{p}) = \frac{\rho}{E_p p_+ p_-} \int \frac{d^3 p'}{(2\pi)^3} \sqrt{\frac{p_+ p'_+}{\nu_s \nu'_s}} \frac{4m\sqrt{\rho\rho'}}{\vec{q}^2 E_p E_{p'}} \chi(\vec{p}') \quad (2.53)$$

which is solved by the (already BS-normalized [19, 27]) wave function

$$\begin{aligned} \chi(\vec{p}) &= iD_0(M_n^{(0)} - i\Gamma, p) \sqrt{\frac{2E_p + M_n^{(0)}}{2M_n^{(0)}(2E_p + M_n^{(0)}) + 2i\Gamma(M_n^{(0)} - 2E_p)(E_p - i\frac{\Gamma}{2})}} E_p \times \\ &\quad \times (2E_p - M_n^{(0)})(2E_p + M_n^{(0)} - 2i\Gamma) \phi(\vec{p}), \end{aligned} \quad (2.54)$$

$$\bar{\chi}(\vec{p}) = \chi(\vec{p}). \quad (2.55)$$

The latter equation is the same as eq. 2.38 if we use real solutions of the Schrödinger equation. It is preferable within the present context because of the nasty complex expressions in 2.54.

2.3 Perturbation Theory

In view of the calculation of the $t\bar{t}$ potential we observe that the BR wave function for decaying fermions is the same as for stable ones, which means that, as far as the systematic determination of the QCD potential is concerned, we may just consider perturbation theory for "nonabelian positronium" for its determination. Perturbation theory for the BS equation starts from the BR equation for the Green function $G_{BR} = G_0$ of the scattering of two fermions [23] which is exactly solvable. D_0 is the product of two zero order propagators, K_0 the corresponding kernel. The exact Green function may be represented as

$$G = \sum_l \chi_{nl}^{BS} \frac{1}{P_0 - P_n} \bar{\chi}_{nl}^{BS} + G_{reg} = G_0 \sum_{\nu=0}^{\infty} (H G_0)^{\nu}, \quad (2.56)$$

where the corrections are contained in the insertions H . Bound state poles $P_n = M_n - i\Gamma_n$ occur, of course, only for $M_n < 2m$. It is easy to show that H can be expressed by the full kernel K and the full propagators D :

$$H = -K + K_0 + iD^{-1} - iD_0^{-1}. \quad (2.57)$$

Since the corrections to the external propagators contribute only to $O(\alpha^5)$ to the energy displacement [24], the perturbation kernel is essentially the negative difference of the exact BS-kernel and of the zero order approximation.

Expanding both sides of equation 2.56 in powers of $P_0 - P_n$, the mass shift is obtained [2]:

$$\Delta M - i\frac{\Gamma}{2} = \langle h_0 \rangle (1 + \langle h_1 \rangle) + \langle h_0 g_1 h_0 \rangle + O(h^3). \quad (2.58)$$

Here the BS-expectation values are defined as e.g.

$$\langle h \rangle \equiv \int \frac{d^4 p}{(2\pi)^4} \int \frac{d^4 p'}{(2\pi)^4} \bar{\chi}_{ij}(p) h_{ii'jj'}(p, p') \chi_{i'j'}(p'), \quad (2.59)$$

We emphasize the four-dimensional p-integrations which correspond to the generic case, rather than the usual three dimensional ones in a completely nonrelativistic expansion. Of course, 2.59 reduces to an ordinary "expectation value" involving $d^3 p$ and $\Phi(\vec{p})$, whenever h does not depend on p_0 and p'_0 .

In 2.58 h_i and g_i represent the expansion coefficients of H and G_0 , respectively, i.e.

$$H = \sum_{n=0}^{\infty} h_n (P_0 - P_n)^n \quad (2.60)$$

$$G_0 = \sum_{n=0}^{\infty} g_n (P_0 - P_n)^{n-1} \quad (2.61)$$

Similar corrections arise for the wave functions [2]:

$$\chi^{(1)} = (g_1 h_0 + \frac{1}{2} \langle h_1 \rangle) \chi^{(0)} \quad (2.62)$$

To illustrate the use of this perturbation theory as well as the new zero order equation for scalar particles presented in the last section we will present here the calculation of the fine structure of two stable scalar particles interacting via a vector particle. Existing calculations [22] rely on a mix of Fouldy-Wouthuysen transformation and the iterated Salpeter perturbation theory. Our present approach is much more transparent and allows in principle the inclusion of any higher order effect in a straightforward manner. To be

definite let us consider the stop–anti-stop system. We will calculate the spectrum up to order α_s^4 .

Since in the zero order equation we have replaced the exact one Coulomb exchange 2.34 by K_0 as given in 2.36 we have now to calculate the contribution of $-K_C + K_0$ to the energy levels. This is shown in fig. 2.3a . With

$$\begin{aligned}\langle\langle -K_c \rangle\rangle &= -4\pi\alpha \int \frac{d^4p}{(2\pi)^4} \frac{d^4p'}{(2\pi)^4} \bar{\chi}(p) \frac{(P_0 + p_0 + p'_0)(P_0 - p_0 - p'_0)}{(\vec{p} - \vec{p}')^2} \chi(p') = \\ &= \left\langle \frac{P_0^2 + 2E_p^2 + 2E_{p'}^2}{4P_0\sqrt{E_p E_{p'}}} \frac{4\pi\alpha}{\vec{q}^2} \right\rangle =\end{aligned}\quad (2.63)$$

$$\begin{aligned}&= \left\langle \left(\frac{4m}{2P_0} - \frac{\sigma_n^2}{2} \right) \frac{4\pi\alpha}{\vec{q}^2} \right\rangle \\ \langle\langle K_0 \rangle\rangle &= -\frac{4m}{2P_0} \left\langle \frac{4\pi\alpha}{\vec{q}^2} \right\rangle\end{aligned}\quad (2.64)$$

we obtain

$$\Delta M_C = \langle\langle -K_c + K_0 \rangle\rangle = -\frac{\sigma_n^2}{2} \left\langle \frac{4\pi\alpha}{\vec{q}^2} \right\rangle = -\frac{m\alpha^4}{16n^4}.\quad (2.65)$$

The fact that the p-integrations are well behaved and the result is of $O(\alpha^4)$ proves the usefulness of our zero order kernel.

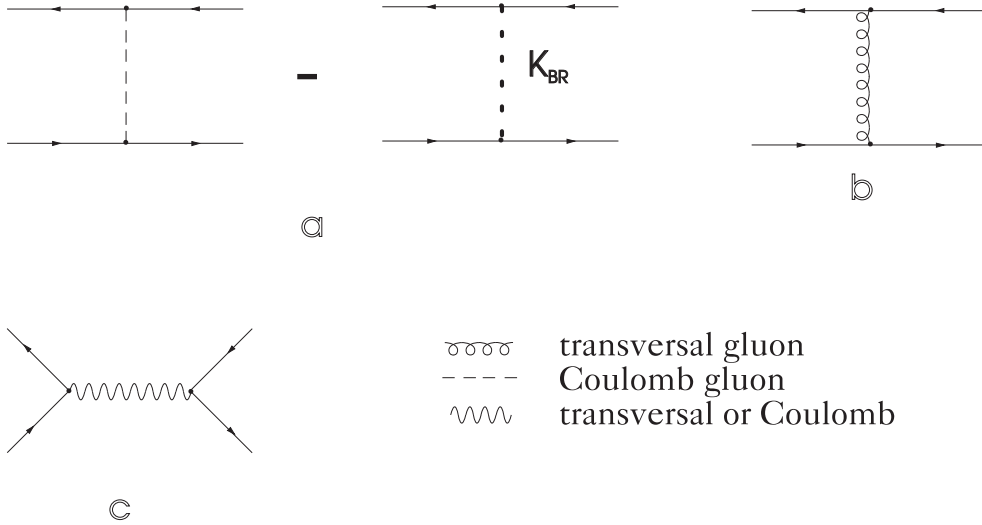


Fig. 2.3

The transverse gluon of fig. 2.3b gives rise to a kernel

$$H_T = \frac{4\pi\alpha}{q^2} \left((\vec{p} + \vec{p}')^2 - \frac{(\vec{p}^2 - \vec{p}'^2)^2}{\vec{q}^2} \right)\quad (2.66)$$

Performing the zero component integrations exactly and expanding in terms of the three momenta one obtains to leading order (c.f. [25])

$$\Delta M_T = \langle \langle H_T \rangle \rangle = \frac{4\pi\alpha}{m^2} \left\langle \frac{\vec{p}^2}{\vec{q}^2} - \frac{(\vec{p}\vec{q})^2}{\vec{q}^4} \right\rangle \quad (2.67)$$

$$= m\alpha^4 \left(\frac{1}{8n^4} + \frac{\delta_{l0}}{8n^3} - \frac{3}{16n^3(l + \frac{1}{2})} \right) \quad (2.68)$$

The annihilation graph into one gluon vanishes due to the color trace since the bound states are color singlets.

The net result for the spectrum of two scalars bound by an abelian gauge field differs from that of ref [22]. The difference can be traced back to the relativistic correction 2.65. We have also checked the derivative $\partial K_0/\partial P_0$ contributing to h_1 and the X-graphs of fig. 2.4.g for possible contributions. Our estimates only yield contributions to higher order.

So far we have calculated the spectrum of two scalars interacting with a abelian gauge field. It has been pointed out long ago [26] that in the case of a nonabelian gauge field further corrections arise due to the gluon splitting vertices. The vertex correction shown below in 2.4.d has been calculated in [26] for the fermionic case.

Here we will give a calculation of the same contribution for scalar constituents. After performing the color trace the perturbation kernel for the second graph in 2.4.e reads

$$H_{2.4.d,2} = -8ig^4 \int \frac{d^4k}{(2\pi)^4} \frac{(P_0 + p_0 + p'_0 - k_0)(-P_0 + p_0 + p'_0)}{\vec{q}^2(\vec{k} - \vec{q})^2[(\frac{P}{2} + p + k)^2 - m^2]k^2} \left(-(\vec{p}\vec{q}) + \frac{(\vec{p}\vec{k})(\vec{q}\vec{k})}{\vec{k}^2} \right). \quad (2.69)$$

Performing the k_0 integration and using the scaling

$$P_0 \rightarrow 2m + O(\alpha^2) \quad (2.70)$$

$$p_0 \rightarrow \alpha^2 p_0 \quad (2.71)$$

$$\vec{k} \rightarrow \alpha \vec{k} \quad (2.72)$$

to extract the leading contribution in α we can show that

$$H_{2.4.d,2} = -\frac{g^2 m}{2} \frac{\vec{p}\vec{q}}{|\vec{q}|^3}. \quad (2.73)$$

Adding the similar contribution from the first graph in 2.4.d gives

$$H_{2.4.d} = -\frac{9\pi^2 \alpha^2 m}{|\vec{q}|}. \quad (2.74)$$

This result differs by a factor $4m^2$ from the fermionic result which is compensated by a corresponding difference in the wave functions to give eventually precisely the same result as in the fermionic case

$$\Delta M = \left\langle \frac{9\pi^2\alpha^2}{4m|\vec{q}|} \right\rangle = \frac{9m\alpha^4}{32n^3(l + \frac{1}{2})}. \quad (2.75)$$

The corrections to the gluon propagator are trivially identical to the fermionic case as can be seen by inspecting the only "critical" step, the p_0 integration 2.63. In view of the fact that the result depends only on the angular momentum and not on the spin this seems reasonable. Thus the difference in the spectrum of the scalar bound state to $O(\alpha^4)$ is entirely due to the tree graphs discussed above.

2.4 The $t\bar{t}$ -Potential

In the last section we showed how the BS-wave function for decaying particles may be reduced to one for stable particles. Therefore, contributions from the width can only arise from graphs with internal fermion lines. We will discuss such a case in detail in sect. 2.5. For the determination of the potential the effect of the width turns out to be negligible. Especially we can extract the contributions to the potential from the level shifts calculated in [27] for the tree and one loop QCD graphs. Our main philosophy will be the following. We estimate the magnitude of an interaction by its effect on the level shift. That is to say our potential to $O(\alpha^4)$ will contain all terms which would give rise to level shifts to that order, keeping in mind that the only level that could be measured is the $1S$ level. With the at present favored top mass value of $\approx 175\text{GeV}$ this would be possible if m_t is known to a sufficient high precision from other experiments by measuring the rising edge of the total cross section $e^+e^- \rightarrow t\bar{t}$. This will be discussed in sect. 3.1. The contribution of the tree and one loop QCD graphs to the potential to $O(\alpha^4)$ reads [28]:

$$\begin{aligned}
\mathcal{V}_1 = & -\frac{\vec{p}^4}{4m^3} - \frac{\alpha\pi}{m^2}\delta(\vec{r}) - \frac{\alpha}{2m^2r} \left(\vec{p}^2 + \frac{\vec{r}(\vec{r}\vec{p})\vec{p}}{r^2} \right) + \\
& + \frac{3\alpha}{2m^2r^3} \vec{L}\vec{S} + \frac{\alpha}{2m^2r^3} \left(\frac{3(\vec{r}\vec{S})^2}{r^2} - \vec{S}^2 \right) + \frac{4\pi\alpha}{3m^2} \vec{S}^2 \delta(\vec{r}) \\
& - \frac{33\alpha^2}{8\pi r} (\gamma + \ln \mu r) + \frac{\alpha^2}{4\pi r} \sum_{j=1}^5 [\text{Ei}(-rm_j e^{\frac{5}{6}}) - \frac{5}{6} + \frac{1}{2} \ln(\frac{\mu^2}{m_j^2} + e^{\frac{5}{3}})] \quad (2.76) \\
& + \frac{9\alpha^2}{8mr^2}.
\end{aligned}$$

In eq. 2.76 the first two lines arise from the relativistic Coulomb correction and the exchange of a transverse gluon. They are, therefore, almost identical to the positronium case if one includes the color factor $4/3$ in the definition of α (c.f eq. 2.8). The only difference is the absence of the annihilation contribution due to the tracelessness of the color matrix at the annihilation vertex. The first term in the third line comes from the gluonic one loop correction to the Coulomb gluon (fig. 2.4.a, 2.4.b). Fermion loops are included with their explicit mass dependence in the next term (fig. 2.4.c). The fourth line contains the contribution of the nonabelian vertex correction fig. 2.4.d. All other graphs shown in fig. 2.4.e-h do not contribute to the required order in Coulomb gauge.

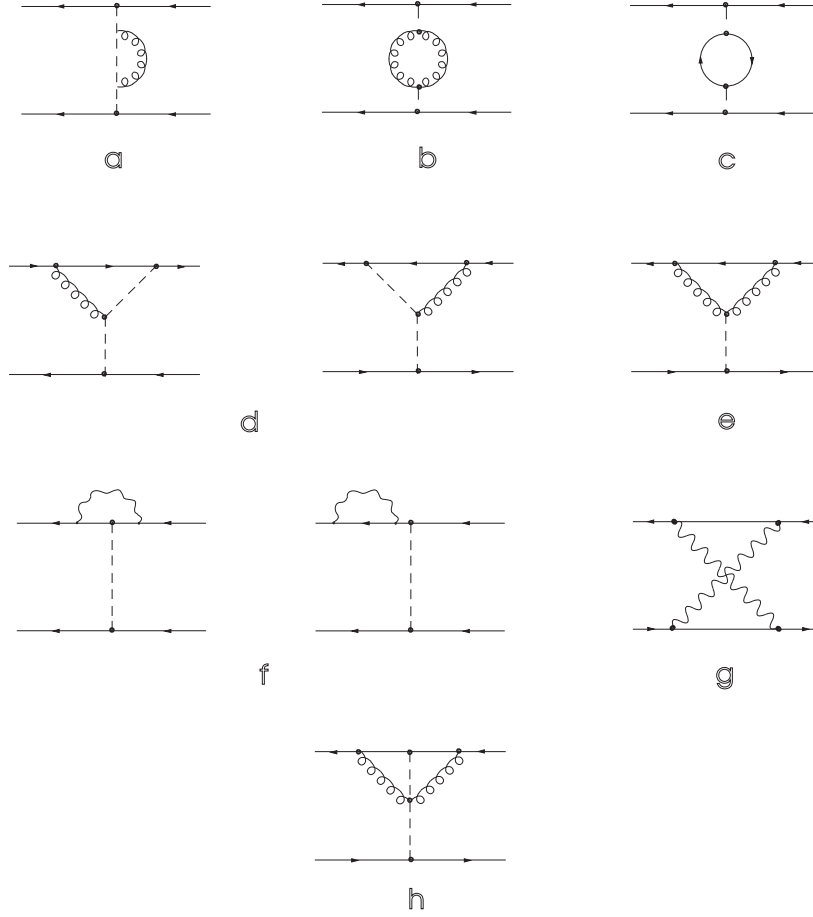


Fig. 2.4

In the following section we will calculate contributions to the potential from two loop QCD graphs and from the electroweak interaction. These contributions will turn out to give effects to the same numerical order of magnitude as the ones above. Therefore, the technique often used to calculate a potential to a given number of loops is shown to be incorrect.

2.4.1 Two Loop Vacuum Polarization

As can be seen from the dependence of the potential 2.76 from the masses of the light flavors, the usual renormalization group arguments relying on massless quarks in the running coupling constant do not consistently include the effect of 'realistic' quark masses in the toponium system, when a systematic BS perturbation is attempted. On the other hand, in a full calculation of effects of $O(\alpha^4)$ two loops with gluons cannot be neglected.

However, in the one loop case finite quark masses yield terms of numerical order $O(\alpha^4)$, therefore it can be expected that in a two loop calculation quark masses will lead to corrections of $O(\alpha^5)$.

Furthermore, since two loop calculations with massive flavors are very cumbersome we circumvent these problems, for the time being, by the following argument, which also includes the three 'massless' quarks u,d,s. Because of the Ward identity for the Coulomb-vertex [3], it is clear from the theory of the renormalization group that the same corrections can be obtained by expanding the running coupling constant with a two loop (gluons+u,d,s) input for the latter which provides also the first 'nonleading' logarithmic contributions. The beta function to two loops is renormalization scheme independent for massless quarks [29] and its two loop part has been calculated some time ago [30]:

$$\beta(g) = -\beta_0 g^3 - \beta_1 g^5 - \dots \quad (2.77)$$

$$\beta_0 = \frac{1}{(4\pi)^2} (11 - \frac{2}{3} n_f) \quad (2.78)$$

$$\beta_1 = \frac{1}{(4\pi)^4} (102 - \frac{38}{3} n_f). \quad (2.79)$$

Here n_f is the number of effectively massless flavors and $\beta(g)$ is the solution of

$$\ln \frac{\sqrt{-q^2}}{\mu} = \int_g^{\bar{g}} \frac{dg'}{\beta(g')}, \quad (2.80)$$

which reads up to two loops

$$\ln \frac{\sqrt{-q^2}}{\mu} = \frac{1}{2\beta_0} \left[\frac{1}{\bar{g}^2} - \frac{1}{g^2} + \frac{\beta_1}{\beta_0} \ln \frac{\bar{g}^2(\beta_0 + \beta_1 g^2)}{g^2(\beta_0 + \beta_1 \bar{g}^2)} \right]. \quad (2.81)$$

Considering this as an equation for $\bar{g} = g(\bar{q}^2)$ we 'undo' the renormalization group improvement by expanding with 'small' $g^2 \propto \alpha$ (cf. eq. 2.8):

$$\begin{aligned} \alpha(\bar{q}^2) = \alpha \left\{ 1 - \alpha \frac{33 - 2n_f}{16\pi} \ln \frac{\bar{q}^2}{\mu^2} + \right. \\ \left. + \frac{\alpha^2}{(16\pi)^2} [(33 - 2n_f)^2 \ln^2 \frac{\bar{q}^2}{\mu^2} - 9(102 - \frac{38}{3} n_f) \ln \frac{\bar{q}^2}{\mu^2}] \right\} \end{aligned} \quad (2.82)$$

Clearly the one loop term agrees with eq. 2.76 in the limit $m_j \rightarrow 0$. That limit, however, is not appropriate here because in this way we would loose terms of numerical $O(m\alpha^4)$. For the computation of the rest we need the expectation value of $(\ln^2 \frac{\bar{q}^2}{\mu^2})/\bar{q}^2$. This integral

can be done analytically (Appendix A) and the result is:

$$\left\langle \frac{\ln^2 \frac{\vec{q}^2}{\mu^2}}{\vec{q}^2} \right\rangle = \frac{m\alpha}{2\pi n^2} \left\{ \frac{\pi^2}{12} + \Psi_2(n+l+1) + s_{nl} + [\Psi_1(n+l+1) + \gamma + \ln \frac{\mu n}{\alpha m}]^2 \right\} \quad (2.83)$$

with

$$s_{nl} = \frac{2(n-l-1)!}{(n+l)!} \sum_{k=0}^{n-l-2} \frac{(2l+1+k)!}{k!(n-l-1-k)^2}.$$

With eq. 2.83 we obtain for the mass shift, induced by the leading logs of the two loop vacuum polarization of the Coulomb gluon a contribution:

$$\begin{aligned} \Delta M_{2loop} = & -\frac{m\alpha^4}{128\pi^2 n^2} \left\{ 27^2 \left[\frac{\pi^2}{12} + \Psi_2(n+l+1) + s_{nl} + (\Psi_1(n+l+1) + \gamma + \ln \frac{\mu n}{\alpha m})^2 \right] + \right. \\ & \left. + 288(\Psi_1(n+l+1) + \gamma + \ln \frac{\mu n}{\alpha m}) \right\}. \end{aligned} \quad (2.84)$$

In this expression we have set $n_f = 3$ as suggested by the number of sufficiently light quarks. Whether eq. 2.84 really represents the full two loop quark- gluon vacuum polarization, numerically consistent with other terms $O(\alpha^4)$, must still be checked in a calculation of the Coulomb gluon's self-energy to two loop order in the Coulomb gauge, i.e. going beyond the sample calculation in [28]. We, nevertheless, indicate the corresponding potential

$$\mathcal{V}_{2loop} = -2 \frac{\alpha^3}{(16\pi)^2 r} \left\{ (33 - 2n_f)^2 \left[\frac{\pi^2}{6} + 2(\gamma + \ln \mu r)^2 \right] + 9(102 - \frac{38}{3}n_f)(\gamma + \ln \mu r) \right\} \quad (2.85)$$

2.4.2 QCD 2-Loop Box Graphs

It would be incorrect to extrapolate from the QED case the absence of corrections to $O(\alpha^4)$, other than the abelian tree graphs because gluon splitting allows new types of graphs. Therefore we will discuss in this section possible sources of new corrections.

Our first example of a QCD box graph is fig. 2.5.a. Between the nonrelativistic projectors λ^\pm of the wave functions 2.25 the perturbation kernel from this graph can be written effectively as

$$\begin{aligned} -iH_{2.5.a} = & 12ig^6 \int \frac{d^4 t}{(2\pi)^4} \frac{d^4 k}{(2\pi)^4} \frac{(p_1^0 - t_0 + m)(p_2^0 - k_0 - m)}{[(p_1 - t)^2 - m^2][(p_2 - k)^2 - m^2]} \times \\ & \times \frac{1}{(t-k)^2 \vec{k}^2 (\vec{q} - \vec{k})^2 \vec{t}^2 (\vec{q} - \vec{t})^2} P(\vec{q}, \vec{k}, \vec{t}) \end{aligned} \quad (2.86)$$

with

$$P(\vec{q}, \vec{k}, \vec{t}) := (\vec{q} - \vec{k})\vec{k} - \frac{[(\vec{t} - \vec{k})(\vec{q} - \vec{k})][\vec{k}(\vec{t} - \vec{k})]}{(\vec{t} - \vec{k})^2}. \quad (2.87)$$

After performing the integrations over the zero components t^0 and k^0 we can use the scaling argument 2.6 to extract the leading contribution. This is justified a posteriori by the (infrared) finiteness of the remaining terms. The resulting expression will thus only depend on \vec{q} :

$$H_{2.5.a} = 6g^6 \int \frac{d^3t}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} \frac{1}{\vec{k}^2(\vec{q} - \vec{k})^2 \vec{t}^2(\vec{q} - \vec{t})^2(\vec{t} - \vec{k})^2} P(\vec{q}, \vec{k}, \vec{t}) \quad (2.88)$$

The trick to simplify this expression is to write the inner products in terms of quadratic expressions that cancel partly the denominator ¹.

$$\begin{aligned} P(\vec{q}, \vec{k}, \vec{t}) = & \frac{1}{4} [2\vec{q}^2 - \vec{k}^2 - (\vec{q} - \vec{k})^2 - \vec{t}^2] \\ & - \frac{\vec{t}^2(\vec{q} - \vec{k})^2}{(\vec{k} - \vec{t})^2} + \frac{\vec{t}^2(\vec{q} - \vec{t})^2}{(\vec{k} - \vec{t})^2} + \frac{\vec{k}^2(\vec{q} - \vec{k})^2}{(\vec{k} - \vec{t})^2} - \frac{\vec{k}^2(\vec{q} - \vec{t})^2}{(\vec{k} - \vec{t})^2} + (\vec{k} - \vec{t})^2 - (\vec{q} - \vec{t})^2] \end{aligned} \quad (2.89)$$

While the original integral 2.88 is finite, the integrals one obtains from a single summand in 2.89 are not. But they are easily evaluated with the help of known results within dimensional regularization [29]. A convenient check of the lengthy computations is that all divergent terms must cancel in the final answer which can be written as

$$H_{2.5.a} = -\frac{81\pi}{128}(12 - \pi^2) \frac{\alpha^3}{|\vec{q}|^2}. \quad (2.90)$$

The nonabelian box graph fig. 2.5.a gives therefore rise to an $O(\alpha^4)$ correction which leads to a slight enhancement of the attractive Coulomb force. This is the only contribution of this type in our renormalization scheme which is defined as follows: m_t is the pole mass, Z_{Gluon} and Z_{1F} are defined by a subtraction at μ . However, the usual predictions for scattering measurements are in the $\overline{\text{MS}}$ scheme. In a calculation of the potential in this scheme a host of contributions of the form of eq. 2.90 are to be expected. Since they are not known yet, and the numerical coefficient in the result 2.90 is small, this at present is more of theoretical than of phenomenological interest. However, it provides an example of a nontrivial contribution to $O(\alpha^4)$ of a two loop graph which certainly is not included e.g. in the running coupling constant. The result 2.90 is new, but a qualitative estimate was mentioned already in [3], [10] and [26].

¹ I am grateful to Antonio Vairo for bringing this trick to my attention

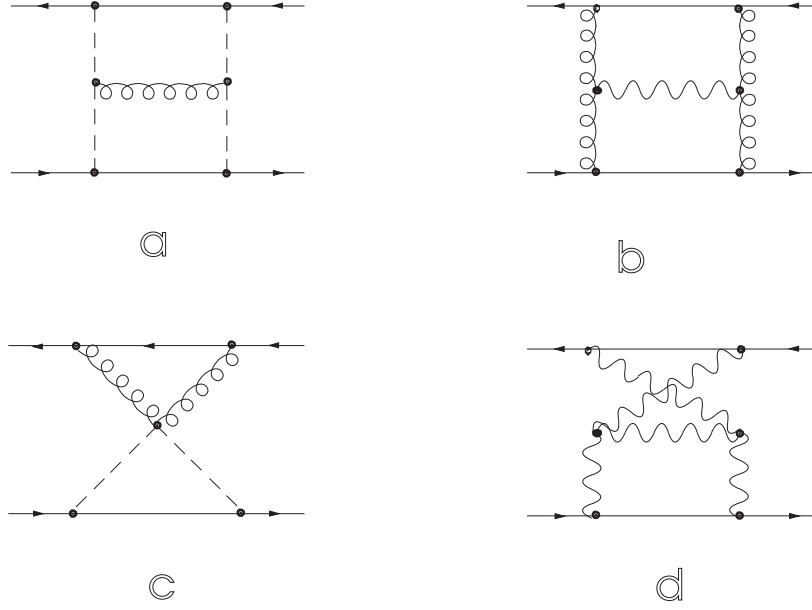


Fig. 2.5

Considering the graph 2.5.b we perform the same steps as for the graph 5.a. One notes that either the spatial gamma matrices at the vertices or the zero component integrations give rise to two additional powers in the spatial momenta and consequently in powers of α . Therefore, this graph contributes only to higher order. It is, however, a spin dependent interaction.

Graphs like fig. 2.5.a and 2.5.b with crossed Coulomb lines (fig. 2.5.d) are irrelevant because they vanish due to group theoretical factors.

By connecting the quark lines between the interactions in graphs 2.4.d and 2.5.a as shown in fig. 2.6 one obtains graphs which could be named nonabelian Lamb shift graphs. We have checked whether these graphs could give rise to further $O(\alpha^4)$ effects - but we did not find any. Thus all these graphs will contribute at the usual Lamb shift level $O(\alpha^5 \ln \alpha)$.

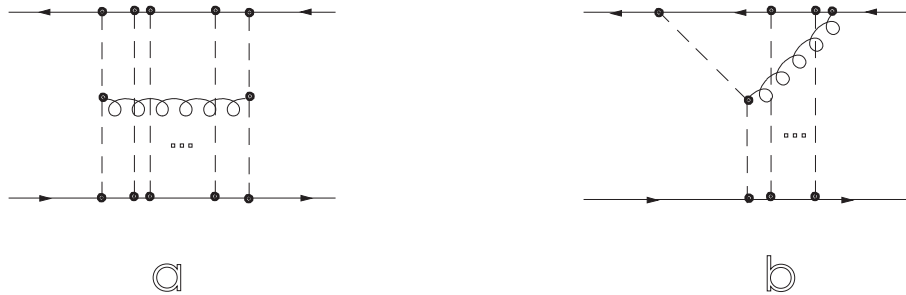


Fig. 2.6

The 'X' graph in fig. 2.5.c can be checked more easily for possible new contributions. As in the calculation of fig. 2.5.a it can be simplified to give

$$\begin{aligned}
 -iH_{2.5.c} &= 3ig^6 \int \frac{d^4t}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{(k_0 + p_1^0 + m)(t^0 + p_2^0 - m)}{[(k + p_1)^2 - m^2][(t + p_2)^2 - m^2] \vec{k}^2 (\vec{q} - \vec{k})^2} \times \\
 &\quad \times \frac{1}{t^2(q - t)^2} \left(1 + \frac{[\vec{t}(\vec{q} - \vec{t})]^2}{\vec{t}^2(\vec{q} - \vec{t})^2} \right). \tag{2.91}
 \end{aligned}$$

The integration over k yields a divergence $1/|\vec{q}|$ if \vec{q} and \vec{p} tend to zero. Contributions within the order of interest may only result from possible poles after the t -integration. For simplicity we consider the part of eq. 2.91 from the factor one in the second line:

$$\begin{aligned}
 I_t &\equiv \int \frac{d^4t}{(2\pi)^4} \frac{t^0 + p_2^0 + m}{[(t + p_2)^2 - m^2]t^2(q - t)^2} = \\
 &= \frac{-i}{(4\pi)^2} \int_0^1 dx \int_0^{1-x} dy \frac{yq^0 - xp_2^0 + p^0 + m}{(yq - xp)^2 - yq^2 - x(p^2 - m^2)} \approx \\
 &\approx \frac{im}{(4\pi)^2} \int_0^1 dx \int_0^{1-x} dy \frac{x}{x^2m^2 + y(1 - x - y)\vec{q}^2} + O(\alpha) = \\
 &= \frac{-i}{(4\pi)^2m} \ln \frac{|\vec{q}|}{m} + O(\alpha). \tag{2.92}
 \end{aligned}$$

Thus the part of graph 2.5.c, specified above, has a leading term from

$$H_{2.5.c,1} = \frac{3g^6}{32(4\pi)^2|\vec{q}|m} \ln \frac{|\vec{q}|}{m}$$

as $\vec{q} \rightarrow 0$ and therefore contributes to $O(\alpha^5 \ln \alpha)$. The second part of graph 2.5.c gives a similar result with a different numerical factor.

We conclude that - opposite to the QED case [31]- box graphs may be important to $O(\alpha^4)$. They could, in fact, be responsible for changes of the zero order coupling of the Coulomb gluon. As a matter of fact, indeed substantially different values of that coupling seem to be required in phenomenological fits of lighter quarkonia.

As illustrated by the explicit calculations above, to $O(\alpha^5)$ beside abelian QED type corrections [24, 31], a host of further non-abelian contributions can be foreseen.

2.4.3 QED Correction

As a rule, the consideration of electromagnetic effects in QCD calculations is not necessary, but at high energies the strong coupling decreases, and in the case of toponium we expect

α_s^2 to be of the same order as α_{QED} . We may obtain this contribution by simply solving the BR equation for the sum of a QED and a QCD Coulomb exchange. This results in the energy levels

$$\begin{aligned} P_0 = M_n^0 &= 2m\sqrt{1 - \frac{(\alpha + \alpha_{QED}Q^2)^2}{4n^2}} \approx \\ &\approx 2m - m\frac{\alpha^2}{4n^2} - \frac{m\alpha\alpha_{QED}Q^2}{2n^2} - m\frac{\alpha^4}{64n^4} - \frac{m\alpha_{QED}^2Q^4}{4n^2} + O(\alpha^6), \end{aligned} \quad (2.93)$$

where Q is the electric charge of the heavy quark, i.e. $2/3$ for toponium. Clearly even the 'leading' third term can only be separated from the effect of the second one to the extent that $\alpha(\mu)$ and $\alpha_{QED}(\mu)$ can be studied separately with sufficient precision.

2.4.4 Weak Corrections

While also weak interactions usually can be neglected in QCD calculations, this is not true in the high energy region, because the weak coupling scales like $\sqrt{G_F m^2}$, becoming comparable to the strong coupling if the fermion mass m is large. Even bound states through Higgs exchange are conceivable [32]. Therefore, we have to consider weak corrections and especially the exchange of a single Higgs or Z particle, assuming for simplicity the standard model with minimal Higgs sector.

The Higgs boson gives rise to the kernel

$$-iH_{Higgs} = -i\sqrt{2}G_F m^2 \frac{1}{q^2 - m_H^2} \approx i\sqrt{2}G_F m^2 \frac{1}{\vec{q}^2 + m_H^2}, \quad (2.94)$$

in an obvious notation.

Since we do not know the ratio $\alpha m/m_H$, which would allow some approximations if that ratio is small, we calculate explicitly the level shifts by transforming into coordinate space. As in Appendix A, we express the Laguerre polynomials in terms of differentiations of the generating function, do the integration and perform the differentiation afterwards to obtain

$$\Delta M_{Higgs} = -m \frac{G_F m^2 \alpha}{4\sqrt{2}\pi} I_{nl}(a_n), \quad (2.95)$$

valid for arbitrary levels and Higgs boson masses, with

$$a_n = \frac{\alpha m}{nm_H}$$

and

$$I_{nl}(a_n) \equiv \frac{a_n^{2l+2}}{n^2(1+a_n)^{2n}} \sum_{k=0}^{n-l-1} \binom{n+l+k}{k} \binom{n-l-1}{k} (a_n^2 - 1)^{n-l-1-k}. \quad (2.96)$$

As an illustration some explicit results for the lowest levels are given in tab.4.

| | | | | | | |
|---------------|---------------------------|-------------------------------------|----------------------------|--|-------------------------------------|----------------------------|
| n | 1 | 2 | 2 | 3 | 3 | 3 |
| l | 0 | 0 | 1 | 0 | 1 | 2 |
| $I_{nl}(a_n)$ | $\frac{a_1^2}{(1+a_1)^2}$ | $\frac{a_2^2(2+a_2^2)}{4(1+a_2)^4}$ | $\frac{a_2^4}{4(1+a_2)^4}$ | $\frac{a_3^2(3+6a_3^2+a_3^4)}{9(1+a_3)^6}$ | $\frac{a_3^4(4+a_3^2)}{9(1+a_3)^6}$ | $\frac{a_3^6}{9(1+a_3)^6}$ |

Tab. 4

It is evident that eq. 2.95 will give a contribution of order $G_F m^2 \alpha^3$ if the Higgs mass is comparable to the mass of the heavy quark and should therefore be taken into account in a consistent treatment of heavy quarkonia spectra and the related potential to numerical order $O(\alpha^4)$. We will thus consider also a term corresponding to 2.94

$$\mathcal{V}_{Higgs} = -\sqrt{2} G_F m^2 \frac{e^{-m_H r}}{4\pi r} \quad (2.97)$$

as a correction in our potential.

Next we consider the contributions of the neutral current, the single Z-exchange and Z-annihilation. For the Z-exchange in the SM we have

$$H_Z = -\sqrt{2} G_F m_Z^2 [\gamma^\mu (v_f - a_f \gamma_5)]_{\sigma\sigma'} \frac{g_{\mu\nu} - \frac{q_\mu q_\nu}{m_Z^2}}{q^2 - m_Z^2} [\gamma^\nu (v_f - a_f \gamma_5)]_{\rho'\rho} \quad (2.98)$$

with

$$v_f = T_3^f - 2Q_f \sin^2 \Theta_w, \quad (2.99)$$

$$a_f = T_3^f, \quad (2.100)$$

where T_3^f is the eigenvalue of the diagonal SU(2) generator for the fermion f . If f is the top quark then $T_3^f = 1/2$. Because we cannot expect αm to be much smaller than m_Z we use the exact expectation value of the Yukawa potential 2.96. q_0^2 may be dropped in the Z-propagator since this provides at most a further correction of numerical $O(\alpha)$:

$$\begin{aligned}
\Delta M_Z &= -\sqrt{2}G_F m_Z^2 \left\langle \frac{a_f^2(3 - 2\vec{S}^2)(1 + \frac{\vec{q}^2}{3m_Z^2}) - v_f^2}{q^2 - m_Z^2} \right\rangle = \\
&= m \frac{G_F m_Z^2 \alpha}{2\sqrt{2}\pi} [a_f^2(1 - \frac{2}{3}\vec{S}^2)(I_{nl}(\frac{\alpha m}{nm_Z}) + \frac{(\alpha m)^2}{2m_Z^2 n^3} \delta_{l0}) - \frac{v_f^2}{2} I_{nl}(\frac{\alpha m}{nm_Z})]
\end{aligned} \tag{2.101}$$

$\vec{S}^2 = S(S+1)$ is total spin of the quark-antiquark system. Therefore this expression gives rise to a singlet-triplet splitting within the order of interest.

Z-annihilation may also yield a sizeable effect. The corresponding energy shift is easily evaluated

$$\Delta M_{S=0} = \frac{3G_F m^2 a_f^2 m \alpha^3}{2\sqrt{2}\pi n^3} \delta_{l0} \tag{2.102}$$

$$\Delta M_{S=1} = -\frac{v_f^2}{a_f^2} \frac{m_Z^2}{4m^2 - m_Z^2} \Delta M_{S=0} \approx 10^{-2} \Delta M_{S=0} \tag{2.103}$$

and also produces a singlet-triplet splitting. It should be noted that the two last contributions yield singlet-triplet splittings which are as important as the usual Breit interaction (cf. the first two lines in 2.76). The corresponding contribution to \mathcal{V} is

$$\begin{aligned}
\mathcal{V}_Z &= \sqrt{2}G_F m_Z^2 a_f^2 \left\{ \frac{e^{-m_Z r}}{2\pi r} \left[1 - \frac{v_f^2}{2a_f^2} - (\vec{S}^2 - 3\frac{(\vec{S}\vec{r})^2}{r^2}) \left(\frac{1}{m_Z r} + \frac{1}{m_Z^2 r^2} \right) - (\vec{S}^2 - \frac{(\vec{S}\vec{r})^2}{r^2}) \right] + \right. \\
&\quad \left. + \frac{\delta(\vec{r})}{m_Z^2} (7 - \frac{11}{3}\vec{S}^2) \right\}
\end{aligned} \tag{2.104}$$

2.4.5 Schwinger-Christ-Lee Terms

As mentioned in chapter 1, nonlocal interactions have to be added to the Lagrangian in Coulomb gauge. We are not aware of any previous attempt to look whether these terms give contributions to bound state problems or to some effective potential.

By analogy to the second ref. [21] we calculate the v_1 term to $O(g^4)$

$$\begin{aligned}
v_1 &= -g^4 \frac{9}{16} \int d^3 r d^3 r' d^3 r'' A_i^c(\vec{r}') K_{ij}(\vec{r} - \vec{r}') K_{jk}(\vec{r} - \vec{r}'') A_k^c(\vec{r}'') \\
K_{ij}(\rho) &= \frac{1}{4\pi|\rho|} \left[\frac{\delta_{ij}}{3} \delta(\vec{\rho}) - \frac{1}{4\pi|\vec{\rho}|^5} (3\rho_i \rho_j - \vec{\rho}^2 \delta_{ij}) \right].
\end{aligned} \tag{2.105}$$

This corrects the gluon propagator by

$$\delta G_{\mu\nu}^{ab}(x_1, x_2) = -\frac{1}{Z[0]} \frac{\delta^2}{\delta J_\mu^a(x_1) \delta J_\nu^b(x_2)} \frac{9ig^4}{16} \int d^4 x \int d^3 r d^3 r' d^3 r'' \frac{\delta}{\delta J_i^c} K_{ij} K_{jk} \frac{\delta}{\delta J_k^c} Z_0[J]. \tag{2.106}$$

In momentum space δG can be calculated by using dimensional regularization to give

$$\delta G_{mn}^{ab}(q, q') = (2\pi)^4 i \delta^{ab} \delta(q - q') \frac{9g^4 \bar{q}^2}{8^5} \frac{1}{q^2 \bar{q}^2} (-\delta_{mn} + \frac{q_m q_n}{\bar{q}^2}), \quad (2.107)$$

which means that we have a mass shift with the same structure as the one transverse gluon exchange (cf. [28]), further suppressed by two more orders in α .

Since the second term v_2 also represents a correction to the propagator of the transverse gluon, it can be estimated by the same method to contribute only in higher orders of α as well. We thus find that both terms can be neglected even including terms of $O(\alpha^5)$.

Finally we will collect the results obtained in this section. Both, the large mass and the large width of the top quark provide a new field for rigorous QCD perturbation theory: In contrast to the lighter quarkonia, (unstable) toponium is a weakly bound system, to be treated by Bethe-Salpeter methods in a systematic manner. The large width even further reduces the effects of confinement. As shown first by Fadin and Khoze [35] Γ can be included in the (weakly bound) Green function at the threshold in a straightforward manner by analytic continuation of the total energy into the complex plane.

We first showed that a similar trick may be also applied to a different zero order equation, the BR-equation. On the basis of this result we describe how to obtain the proper potential \mathcal{V} for such a Green function, rigorously derived from QCD in a perturbative sense. Although a fully 4-dimensional formalism is used, which especially also allows the inclusion of off-shell effects, our result allows an interpretation as a correction to the Coulomb potential:

$$\mathcal{V} = \mathcal{V}_1 + \mathcal{V}_2 \quad (2.108)$$

with \mathcal{V}_1 from eq. 2.76 and

$$\begin{aligned} \mathcal{V}_2 = & -\frac{81}{128} \left(3 - \frac{\pi^2}{4}\right) \frac{\alpha^3}{r} - \frac{2\alpha^3}{(16\pi)^2 r} \left\{ 27^2 \left[\frac{\pi^2}{6} + 2(\gamma + \ln \mu r)^2 \right] + 576(\gamma + \ln \mu r) \right\} \\ & - \frac{8}{9} \frac{4\pi\alpha_{QED}(\mu)\alpha}{r} - \sqrt{2} G_F m^2 \frac{e^{-m_H r}}{4\pi r} + \sqrt{2} G_F m_Z^2 a_f^2 \frac{\delta(\vec{r})}{m_Z^2} \left(7 - \frac{11}{3} \vec{S}^2\right) \\ & + \sqrt{2} G_F m_Z^2 a_f^2 \frac{e^{-m_Z r}}{2\pi r} \left[1 - \frac{v_f^2}{2a_f^2} - (\vec{S}^2 - 3 \frac{(\vec{S}\vec{r})^2}{r^2}) \left(\frac{1}{m_Z r} + \frac{1}{m_Z^2 r^2} \right) - \left(\vec{S}^2 - \frac{(\vec{S}\vec{r})^2}{r^2} \right) \right] \end{aligned} \quad (2.109)$$

It has been obtained by checking contributions up to numerical order $O(\alpha^4)$ to real energy shifts, calculated independently from the weak decay $t \rightarrow b + W$. The abelian relativistic corrections and the one loop contributions are collected in \mathcal{V}_1 .

The first term of \mathcal{V}_2 is the nonabelian box contribution, followed by the two loop corrections to the coulomb gluon propagator. Here the effects of the quark masses are not known yet. On the other hand, we show that certain electroweak effects (QED,Higgs,Z) are numerically important to $O(\alpha^4)$. Their respective contributions are listed in the last two lines of 2.109.

Within a rigorous field-theoretical philosophy it would be incorrect to add, say, a linear term to 2.76 in order to describe confinement. At best 2.76 could be supplemented by a piece $\propto \langle G^2 \rangle r^3$ which mimics the tail of confinement effects by gluon condensate [7, 36].

In the derivation of our potential we have not only used the level shifts, but also have described in much detail new closed forms for such shifts etc. The reason for that has been that on the one hand we hope to have given new useful methods to be applicable also for future treatments of nonabelian $O(\alpha^5)$ -effects. On the other hand certain computations of level shifts may be useful in conjunction with semi-phenomenological approaches to the lighter quarkonia. However, even in the case of the bottom quark the nonperturbative corrections become quite large and their perturbative inclusion seems rather dangerous.

2.5 Relativistic and Gauge Independent Off-Shell Corrections to the Toponium Decay Width

As already mentioned in the previous chapters, the top quark decays almost exclusively into W^+b . Clearly the tree level SM decay width has to be corrected by QCD and weak corrections. QCD corrections to $O(\alpha_s)$ [37] and the leading electroweak corrections [38] have already been calculated. The width is also strongly influenced by supersymmetric corrections [39]. For all these aforementioned corrections it suffices to consider the t -quark as a free on-shell particle since it is off-shell near threshold only to $O(\alpha_s^2)$. This means also that near threshold not only the $O(\alpha_s^2)$ corrections to the free decay are needed but also bound state corrections to the decay width play a role. The calculation of these corrections will be the subject of this chapter. Concerning the $O(\alpha_s^2)$ corrections to the free decay only a partial result is known yet [40].

2.5.1 Narrow Width Approximation

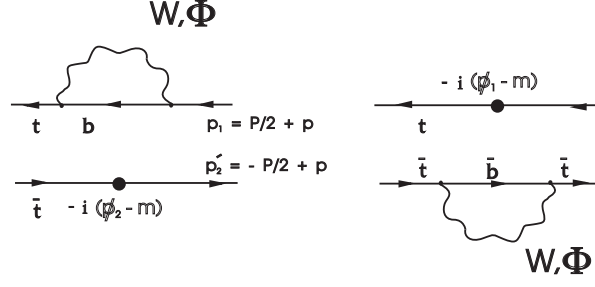
In this section we will calculate off-shell and relativistic bound-state corrections for the decay $t \rightarrow b + W$ to $O(\alpha_s^2)$ making full use of the Bethe-Salpeter formalism for weakly bound systems [41]. As a first step, we will use the original BR equation as zero order equation. Therefore the calculations in this section are performed in the narrow width approximation. However, we will convince ourselves in the next sections by using the BR equation for decaying particles developed in sect. 2.1 that this actually gives the correct result.

We are able to take into account all terms to that order in a systematic and straightforward manner. One of the previously not considered contributions is obtained by explicit calculation and cancels precisely gauge dependent terms which appeared in an earlier [45] off-shell calculation. We will see that important cancellations also determine the gauge-independent part.

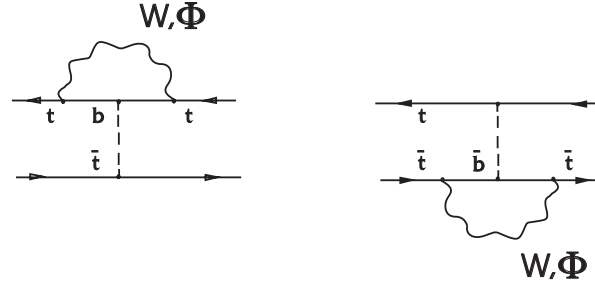
As proposed first in ref. [42] the bound state effects for the inclusive reaction $\sigma_{e^+e^- \rightarrow t\bar{t}}$ near the $t\bar{t}$ threshold could be taken into account by including the width Γ_0 for a freely decaying top quark [43] in the absorptive part of the vacuum polarization, through the replacement of the total energy E by $E + i\Gamma_0$. Subsequently this approach was applied in other detailed numerical studies involving potentials with the running strong coupling constant for the short distance Coulomb term and also including a confining part [44,

45, 46]. However, as noted recently in [45], the introduction of a constant decay rate Γ_0 in the Breit-Wigner resonance part of this formalism leads to problems with unitarity. Therefore a "running" width $\Gamma(p^2)$ for a virtual top was proposed with $p^2 \neq m_t^2$. As expected, a naive off-shell extrapolation of Γ_0 leads to ambiguous and gauge dependent results. $\Gamma(p^2)$ was determined indirectly from the vacuum polarization function making certain approximations [45]. Furthermore it was claimed that the running width would give rise to effects of the order of 10% to the total cross section. *Qualitatively* the main physical effects from t decaying in toponium seem to be quite well understood by now [47]: On the one hand, Γ decreases by time dilatation and by reduction of phase space, caused by the decay below the mass shell. Cancellations should occur with an increase of Γ due to the Coulomb-enhancement induced by the Coulomb interaction of the relativistic b -quark, because this overlap effect is known to be important from the qualitatively similar muonium system [49]. The purpose of this chapter is to show that existing quantum field theoretical technology, well tested e.g. in the (abelian) positronium case, may be used also here to solve the problem of a gauge-independent "running width", including the qualitative effects listed above. Even the Coulomb enhancement can be taken into account properly in this context, which to the best of our knowledge has not been possible before.

The correction to the decay width is obtained from the imaginary part of the expectation values, receiving contributions from the graphs in fig.2.7.



a: Weak off-shell corrections (Σ)



b: Coulomb enhancement (Λ)

Fig. 2.7

It is important to note that eq. 2.58 is not a power series in α_s . In 2.58 it will be essential that $O(\alpha_s^2)$ corrections arise from all three terms denoted here if the BR wave-functions are used in lowest order. It should be emphasized that starting from the Schrödinger equation instead, the inclusion of the proper relativistic corrections to the free fermion propagators would have necessitated the inclusion of complicated higher order terms in h . We also anticipate that in the present application of this formalism for the imaginary part of 2.58, $-\Gamma/2$, renormalization effects need not be considered.

The graph fig.2.7.a gives rise to the BS perturbation kernel

$$H^\Sigma = i[\Sigma(p_1) \otimes (\not{p}_2 - m) + (\not{p}_1 - m) \otimes \Sigma(p_2)](2\pi)^4 \delta(p - p') \quad (2.110)$$

where $p_1 = \frac{P_0}{2} + p$ and $-p_2 = \frac{P_0}{2} - p'$ denote the four momentum of the quark and the antiquark, respectively. The direct product refers to the $t \otimes \bar{t}$ spinor space. The factors $(\not{p}_1 - m)$ and $(\not{p}_2 - m)$ compensate the superfluous propagator on the line without Σ .

Since we are calculating higher order effects we neglect the mass of the bottom quark although it may be included in principle. For the electroweak theory we use the R_ξ -gauge

with the gauge fixing Lagrangian (M denotes the mass of the W-Boson)

$$\mathcal{L}_{gf} = -\xi |\partial^\mu W_\mu^+ - i \frac{M}{\xi} \phi^+|^2$$

because it eliminates mixed $W - \phi$ propagators. The gauge parameter ξ will not be fixed in the following. We obtain for the imaginary parts of Σ in 2.110 ($s = \sin \theta_W$, $\mathcal{P}_\pm = (1 \pm \gamma_5)/2$) :

$$-\text{Im}\Sigma = \frac{e^2}{s^2} \frac{\not{p}}{16\pi} \Theta(p^2 - M^2) [\mathcal{P}_+ A^{(W)} + \mathcal{P}_- A^{(\phi)}] \quad (2.111)$$

where (for $p^2 > M^2, \xi > M^2/p^2$)

$$\begin{aligned} A^{(W)} &= \tilde{t}(p^2) + \rho(p^2, \xi) \\ A^{(\phi)} &= m^2 \left(\frac{\tilde{t}(p^2)}{2M^2} - \frac{\rho(p^2, \xi)}{p^2} \right) \\ \tilde{t}(p^2) &= \frac{(p^2 - M^2)^2}{2p^4}, \quad \rho(p^2, \xi) = \frac{1 - \xi}{2\xi} \left(1 - \frac{M^2}{2p^2} \frac{1 + \xi}{\xi} \right) \end{aligned} \quad (2.112)$$

In evaluating the expectation value according to the rule 2.59 we encounter for both terms the same trace (cf 2.11 and 2.23)

$$\begin{aligned} T &:= \frac{1}{2} \text{tr} [\gamma_0 \Lambda^- S^\dagger \Lambda^+ \gamma_0 \mathcal{P}_\pm \not{p}_1 \Lambda^+ S \Lambda^- (\not{p}_2 - m)] = \\ &= \frac{1}{2} (p_0 + \omega) [m(1 - \frac{\vec{p}^2}{m^2}) + (p_0 - \omega)] + O(\alpha_s^4). \end{aligned} \quad (2.113)$$

It is seen to gives rise to (spin independent) relativistic corrections of the same order as the off-shell ones from Σ . Inserting 2.112 into 2.111 we may expand in powers of $(p^2 - m^2)$, because terms of order $(p^2 - m^2)^2$ already yield negligible orders in α_s :

$$\begin{aligned} -\text{Im}\Sigma^{(W)} &= \frac{e^2 m}{32\pi s^2} \left[\left(2 + \frac{m^2}{M^2} \right) \tilde{t}(m^2) + C \frac{p^2 - m^2}{m^2} \right] + O((p^2 - m^2)^2) \\ C &= \left(2 + \frac{m^2}{M^2} \right) \sqrt{2\tilde{t}(m^2)} \frac{M^2}{m^2} + 2\rho(m^2, \xi) \end{aligned} \quad (2.114)$$

Here $\text{Im}\Sigma^{(W)}$ denotes $\text{Im}\Sigma$ with $\not{p} \rightarrow m$ and without \mathcal{P}_\pm . Thus the total contribution of fig.2.7.a to the decay width becomes ($h_0^\Sigma = H^\Sigma|_{P_0=M_{n,0}}$)

$$\begin{aligned} \Gamma_1 &= -2\text{Im}\langle h_0^\Sigma \rangle = -2\langle \left(1 - \frac{5\vec{p}^2}{4m^2} - \frac{3}{4}\sigma_n^2 \right) \text{Im}\Sigma \rangle = \\ &= \Gamma_0 - 2\Gamma_0 \sigma_n^2 + \frac{e^2 m}{16\pi s^2} C \langle \frac{p^2 - m^2}{m^2} \rangle, \end{aligned} \quad (2.115)$$

where Γ_0 is the well known zero order result which is twice the decay width of a free top quark [42, 43]. A factor 2 arises here and in the following, counting both self energy contributions of the t and \bar{t} line:

$$\Gamma_0 = -2\text{Im}\Sigma^{(W)}(m) = \frac{e^2 m}{16\pi s^2} \left(1 + \frac{m^2}{2M^2}\right) \frac{(m^2 - M^2)^2}{m^4} \quad (2.116)$$

The expectation value $\langle \frac{p^2 - m^2}{m^2} \rangle$, left over after the k_0 integration, is taken in the ordinary sense, i.e. between the (normalized) Coulomb wave functions $\phi_{nlm}(\vec{k})$ in 2.23.

In the graphs shown in fig. 2.7.b to the required order it proves sufficient to evaluate the leading contribution by applying the nonrelativistic wave functions 2.25. After a straightforward but tedious calculation we obtain for the imaginary part of the vertex function $\Lambda_0^{(W)}$ ($\xi > M^2/m^2$ and $1 > M^2/m^2$):

$$\begin{aligned} \text{Im}\Lambda_0^{(W)} &= \frac{e^2}{32\pi s^2} \gamma_0 F \\ F &= \left(1 + \frac{m^2}{2M^2}\right) \left(1 - \frac{M^2}{m^2}\right) \left(1 + 3\frac{M^2}{m^2}\right) + 4\rho(m^2, \xi). \end{aligned} \quad (2.117)$$

In the BS expectation value corresponding to the first term on the r.h.s of 2.58 the k_0 integration to this order allows the simple replacement $k_0 \rightarrow -\omega$. With the spinor trace $(-\frac{1}{2})$ after including the nonrelativistic projectors in 2.25 we arrive with $\text{Im}\Lambda_0^{(W)}$ at $(\vec{q} = \vec{p} - \vec{p}')$

$$\Gamma_2 = \frac{e^2}{16\pi s^2} \langle \frac{4\pi\alpha}{\vec{q}^2} \rangle F. \quad (2.118)$$

Remembering that $\vec{p} = O(\alpha_s)$, the last expectation value of eq. 2.115 can be rewritten

$$\langle \frac{p^2 - m^2}{m^2} \rangle \approx -2\langle \sigma^2 + \frac{\vec{p}^2}{m^2} \rangle = -\langle \frac{2}{m} \frac{4\pi\alpha}{\vec{q}^2} \rangle, \quad (2.119)$$

where the Schrödinger equation with reduced mass $m/2$ has been used in the last equality. We note already here that with eq. 2.119 the gauge dependent terms proportional $\rho(m^2, \xi)$ exactly cancel in 2.115 plus 2.118!

So far only the first term in 2.58 has been considered. The second order terms in \hbar for the leading correction $O(\alpha_s^2\alpha)$ can be summarized as

$$\Gamma_3 = -2\text{Im}[\langle \langle h_0^{QCD} g_1 h_0^\Sigma \rangle \rangle + \langle \langle h_0^\Sigma g_1 h_0^{QCD} \rangle \rangle] \quad (2.120)$$

$$\Gamma_4 = -2\text{Im}[\langle \langle h_0^{QCD} \rangle \rangle \langle \langle h_1^\Sigma \rangle \rangle + \langle \langle h_0^\Sigma \rangle \rangle \langle \langle h_1^{QCD} \rangle \rangle] \quad (2.121)$$

For g_1 (cf. eq. 2.61) in 2.120 we find it convenient to write

$$g_1 := \left[G_0 - \frac{\bar{\chi} \otimes \chi}{P_0 - M_{n,0}} \right]_{P_0=M_{n,0}} \quad (2.122)$$

We first evaluate the contribution $\Gamma_3^{(1)}$ of the Green function G_0 expanded around $P_0 - M_n = 2m\sqrt{1 - \sigma^2} - 2m\sqrt{1 - \sigma_n^2} \approx m(\sigma_n - \sigma^2)$. For our purpose it suffices to calculate the leading nonrelativistic contribution since relativistic corrections are suppressed by an additional factor $O(\alpha_s^2)$. Therefore we can use the nonrelativistic Green function $G_{nr}(k'', k', \sigma)$, i.e. essentially Schwinger's solution $G_{nr}(\vec{k}'', \vec{k}', \sigma)$ to the Coulomb problem [21], supplemented by appropriate factors $(k_0^2 - \omega^2)^{-1}$ so as to include also the zero component of relative momentum. For $P_0 \neq M_{n,0}$ and thus $\sigma \neq \sigma_n$ the first term of eq. 2.122 yields a contribution (cf. 2.25)

$$\Gamma_3^{(1)} = -4 \int dk dk' dk'' \bar{\chi}_{nr}(k'')(k'' + \omega_n'')(i\text{Im}\Sigma) G_{nr}(k'', k', \sigma) h_0^{QCD}(k', k) \chi_{nr}(k) \quad (2.123)$$

where again $-(k'' + \omega_n'')$ represents the inverse nonrelativistic propagator in fig. 2.7.a. Carrying out the k_0 integrations leads to $(d\vec{k} := d^3k/(2\pi)^3 \text{ etc. })$

$$\Gamma_3^{(1)} = -2\Gamma_0 \int d\vec{k} d\vec{k}' d\vec{k}'' \Phi^*(\vec{k}'') \left[1 - \frac{m(\sigma^2 - \sigma_n^2)}{4\omega''} \right] G_{nr}(\vec{k}'', \vec{k}', \sigma) h_0^{QCD}(\vec{k}', \vec{k}) \Phi(\vec{k}), \quad (2.124)$$

if we use the fact that for h_0^{QCD} to $O(\alpha_s^2)$ the QCD corrections in H from $K_C - K_{BR}$, Coulomb gluon exchange etc. are real and independent of k_0 .

The factor proportional to 1 from the square bracket in 2.124 exactly reproduces the second part of 2.122, to be subtracted out before the limit $\sigma \rightarrow \sigma_n$ is taken. The limit $(\sigma^2 - \sigma_n^2) \rightarrow 0$ just selects the corresponding pole term $\phi_n^* \phi_n [\frac{m}{2}(\sigma^2 - \sigma_n^2)]^{-1}$ in $G_{nr}(\vec{k}'', \vec{k}', \sigma)$. Thus from 2.120 only

$$\Gamma_3 = \frac{\Gamma_0}{2} \langle h_0^{QCD} \rangle \langle \frac{1}{\omega} \rangle \quad (2.125)$$

remains. $\langle h_0^{QCD} \rangle$ could be taken from refs. [28, 26]. Its explicit form will turn out not to be relevant here.

For the first term of 2.121 h_1^Σ , i.e. the derivative of 2.110 with respect to P_0 at $P_0 = M_{n,0}$ is needed. Again the factor $\langle h_0^{QCD} \rangle$ provides additional powers of α_s and hence allows the use of the mass shell condition $P_0 \rightarrow 2m$ in

$$h_1^\Sigma = (2\pi)^4 i \delta(p - p') [\gamma_0 \Sigma \otimes (\frac{1}{2} \gamma_0) + (\frac{1}{2} \gamma_0) \otimes \gamma_0 \Sigma]. \quad (2.126)$$

Thus from this term

$$\Gamma_4^{(1)} = -2\langle\langle h_0^{QCD} \rangle\rangle \text{Im}\langle\langle h_1^\Sigma \rangle\rangle \approx -\frac{\Gamma_0}{2}\langle h_0^{QCD} \rangle \langle \frac{1}{\omega} \rangle \quad (2.127)$$

follows.

In the second part of eq. 2.121 h_1^{QCD} , i.e. the first derivative of H^{QCD} , will only receive contributions from $H = -K_C + K_{BR}$ to $O(\alpha_s^2)$. The instantaneous Coulomb kernel K_C does not depend on P_0 , nor does the vacuum polarization [26] correction which had provided the leading contribution to $\langle h_0^{QCD} \rangle$ above:

$$\Gamma_4^{(2)} = -2[\text{Im}\langle\langle h_0^\Sigma \rangle\rangle] \langle\langle \frac{\partial}{\partial P_0} K_{BR}|_{P_0=M_{n,0}} \rangle\rangle. \quad (2.128)$$

Inserting the explicit (real) expression for the BR kernel K_{BR} , (eq. 2.14 with $\Gamma \rightarrow 0$), we obtain

$$\langle\langle \frac{\partial}{\partial P_0} K_{BR} \rangle\rangle = \int \frac{d^3k}{(2\pi)^3} |\phi(\vec{k})|^2 \frac{P_0 - 2E_k}{2P_0} \approx -\frac{\sigma_n^2}{2} \quad (2.129)$$

and thus

$$\Gamma_4^{(2)} = -\frac{1}{2}\Gamma_0\sigma_n^2. \quad (2.130)$$

Before adding all contributions we note that precisely the contribution Γ_1 in 2.115 has been quoted already in ref. [45] as a result as a specific off-shell extrapolation of the free decay width. It was obtained there by the intuitive replacement $\not{p} \rightarrow \sqrt{p^2}$ in the on-shell expression. However adding Γ_2 of 2.118, with 2.119 *all* gauge dependent terms are found to cancel. Moreover a striking feature of the sum $\Gamma_1 + \Gamma_2$ is that even the *gauge-independent* terms precisely sum to zero as well, which is by no means "natural" in view of the explicit form of F and C , as compared to Γ_0 of 2.116 occuring in the second term of 2.115. In a similar manner also $\Gamma_3 + \Gamma_4^{(1)}$ cancel each other (cf. eq. 2.125 and 2.127), leaving only $\Gamma_4^{(2)}$ in the final result

$$\Gamma_{\text{boundstate}} = \Gamma_0(1 - \frac{1}{2}\sigma_n^2 + O(\alpha_s^3)). \quad (2.131)$$

Recalling that $1 - \sigma_n^2/2 \approx (1 - \langle \vec{k}^2/m^2 \rangle)^{1/2}$, the residual effect may be interpreted as an approximation of the γ -factor for the time dilatation for the weakly bound top. The origin of that term, the relativistically generalized Coulomb-kernel K_{BR} gives some credit to this interpretation. For the complete cancellation of the other terms we will present a convincing explanation in the next section.

Among the cancelled terms the other expected effects can be more or less clearly isolated: Γ_1 (fig. 2.7.a) includes the off-shell corrections and the decrease due to the phase space reduction. But this is here completely cancelled by the "Coulomb enhancement" (fig. 2.7.b). It should be emphasized, though, that bound-state effects of the $b\bar{t}$ or $b\bar{b}$ systems are, of course, irrelevant for total energies near the $t\bar{t}$ threshold. Our systematic perturbative approach also correctly describes the toponium decay to $O(\alpha_{weak})$ as a sum of the decays widths $t \rightarrow b + W^+$ and $\bar{t} \rightarrow \bar{b} + W^-$ (cf. the sum of the terms in fig.2.7).

A reader used to phenomenological calculations may wonder why no running coupling in α_s or even a more "realistic" potential involving a confining piece has been used. The simple answer is that any generalization of this type could in principle completely ruin the strictly perturbative approach (and also the built-in gauge-independence) advocated here, where e.g. the gluon vacuum polarization (cf. h_0^{QCD} above) must be accounted for as a separate perturbation and must under no circumstances be mixed with higher order leading logs in perturbation theory, as summarized in a running α_s . However, we will show in the next section, that this limitation can be circumvented in the case at hand.

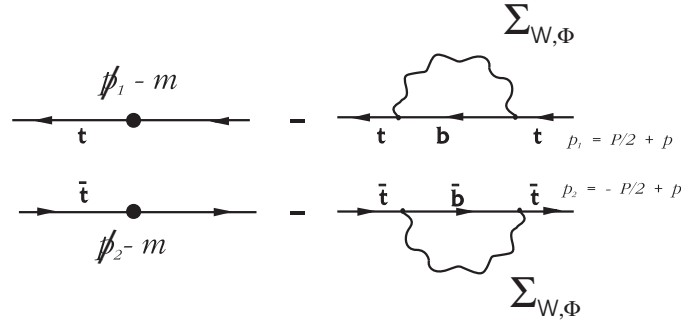
Anyhow, since the real part of the weak bound-state corrections can be estimated to be small, the net effect of our result 2.131 for the discrete levels of toponium allows a simple interpretation: It consists of the (level-independent) free decay width Γ_0 and a very small imaginary correction factor $1 + i\Gamma_0/2m$ to the Rydberg energy.

2.5.2 Including the Width in the Zero Order Equation

In this chapter the bound state problem for a fermion-antifermion system is considered including a finite decay width of the constituents already in the zero order equation as outlined in chapter 2. We focus especially on the $t\bar{t}$ system for which we reconsider the explicit calculation of the bound state corrections to the toponium width of the last section, which was performed in the narrow width approximation and needed the use of second order Bethe-Salpeter perturbation theory. We find that one obtains the same result already in first order BS perturbation theory if one uses our present approach. This also explicitly shows the dependence of any result at a fixed order of BS perturbation theory on the chosen zero order equation.

Furthermore the extensive cancellations of gauge dependent terms turn out to be a consequence of a Ward identity. This cancellation mechanism is shown to be valid for general fermion-antifermion systems in the next section.

The correction to the decay width is obtained from the imaginary part of the expectation values, receiving now contributions from the graphs in fig.2.8 subtracted by the zero order propagator and from fig. 2.7.b.



Weak off-shell corrections (D^1)

Fig. 2.8

Corrections to the fermion propagator have to be included via the perturbation kernel

$$h_0^{(1)} = (iD^{-1} - iD_0^{-1})|_{P_0=M_n-i\Gamma}. \quad (2.132)$$

We emphasize that the perturbation kernel now has to be evaluated at $P_0 = M_n - i\Gamma$. With eq. 2.57 and $D_0^{-1}|_{P_0=M_n-i\Gamma} = D_0^{-1}|_{\Gamma=0}$ we find

$$h_0^{(1)} = -i[(\Sigma(p_1) + i\frac{\Gamma}{2}\gamma_0) \otimes (\not{p}_2 - m) + (\not{p}_1 - m) \otimes (\Sigma(p_2) - i\frac{\Gamma}{2}\gamma_0)](2\pi)^4\delta(p - p') \quad (2.133)$$

where $p_1 = \frac{P_0}{2} + p$ and $-p_2 = \frac{P_0}{2} - p$ denote the four momentum of the quark and the antiquark, respectively. The direct product refers to the $t \otimes \bar{t}$ spinor space. The factors $(\not{p} - m)$ and $(\not{p}' - m)$ compensate the superfluous propagator on the line without Σ . We have neglected terms that will contribute to $O(\alpha^2 \Gamma^2/m)$.

Now we observe that the terms in 2.133 which include Σ have already been calculated in the last section except for the imaginary parts occurring in the trace

$$\begin{aligned} T &:= \frac{1}{2} \text{tr}[\gamma_0 \Lambda^- S^\dagger \Lambda^+ \gamma_0 \mathcal{P}_\pm \not{p}_1 \Lambda^+ S \Lambda^- (\not{p}_2 - m)] = \\ &\approx \frac{1}{2} (p_0 + \omega) [m(1 - \frac{\vec{p}^2}{m^2}) + (p_0 - \omega) - i\frac{\Gamma}{2}]. \end{aligned} \quad (2.134)$$

and in

$$p^2 - m^2 \approx 2mp_0 - \vec{p}^2 - m\sigma^2 - im\Gamma. \quad (2.135)$$

We will be able to neglect these imaginary parts in the following since they either give rise to real corrections (which we are not interested in) or to imaginary ones from $\text{Re}\Sigma$, both of which are of $O(\alpha_{weak}^2)$. Of course, these corrections would be required in a complete calculation of the toponium decay width to this order in order to obtain a gauge independent result. This goes beyond the scope of the present work. Furthermore, these corrections are state independent, expected to be very small and therefore of minor practical interest, anyhow. As can be seen from 2.134 the axial part of Σ does not contribute within the expectation value.

Therefore, the contribution from first order BS perturbation theory to the decay width to $O(\alpha_s^2 \Gamma)$ reads

$$\langle \langle h_0^{(1)} \rangle \rangle = \Gamma_1 + \Gamma_3 \quad (2.136)$$

with Γ_1 as defined before

$$\begin{aligned} \Gamma_1 &= -4\text{Im} \langle \langle -i\Sigma(p_1) \otimes (\not{p}_2 - m)(2\pi)^4 \delta(p - p') \rangle \rangle = \\ &= \Gamma_0 - 2\Gamma_0 \sigma_n^2 + \frac{e^2 m}{16\pi s^2} [(2M^2 + m^2) \frac{m^2 - M^2}{m^4} + 2\rho(m^2, \xi)] \langle \frac{p_1^2 - m^2}{m^2} \rangle, \end{aligned} \quad (2.137)$$

and a new contribution from the inverse zero order propagator

$$\Gamma_3 = -4\text{Im} \langle \langle \frac{\Gamma}{2} \otimes (\not{p}_2 - m)(2\pi)^4 \delta(p - p') \rangle \rangle = -\Gamma_0 (1 + \frac{\sigma_n^2}{2}) \quad (2.138)$$

Because of the inclusion of Γ in the zero order equation now in the sum $\Gamma_1 + \Gamma_3$ the leading Γ_0 terms cancel and the resulting correction is already of $O(\alpha_s^2 \Gamma_0)$.

Now we turn to the calculation of the vertex correction (fig. 2.7.b). This correction remains unchanged. However, in sect. 2.5.1 we calculated $\text{Im}\Lambda_0 := \lim_{q \rightarrow 0, p^2 \rightarrow m^2} \Lambda(p, q)$ directly. Here we will show that it is possible to use a Ward identity to determine this correction. This also provides a good check for our explicit calculation.

We observe that

$$\Lambda_0^a(p, q) \Big|_{q \rightarrow 0} = -g_{QCD} T^a \frac{\partial}{\partial p_0} \Sigma(p) \quad (2.139)$$

holds at least at the one-loop level. This identity is believed to hold to all orders if one takes into account the equivalence of Coulomb gauge calculations in the usual and in the background field method and the Ward identities derived in the latter approach [50].

In the graphs shown in fig. 2.7.b to the required order it proves sufficient to evaluate the leading contribution by putting the fermions on the mass shell and applying the nonrelativistic wave functions 2.25. Using the explicit expression 2.111 for $\text{Im}\Sigma$ we obtain with the help of eq. 2.139

$$\begin{aligned} \text{Im}\Lambda_0 &= \frac{e^2}{s^2 16\pi} \gamma_0 [A^{(W)}(m^2) + A^{(\phi)}(m^2) + 2m^2 \frac{\partial}{\partial p^2} (A^{(W)}(p^2) + A^{(\phi)}(p^2)) \Big|_{p^2=m^2}] \\ &= \frac{e^2}{32\pi s^2} \gamma_0 F \\ F &= (1 + \frac{m^2}{2M^2})(1 - \frac{M^2}{m^2})(1 + 3\frac{M^2}{m^2}) + 4\rho(m^2, \xi). \end{aligned} \quad (2.140)$$

This result agrees with that of 2.117 and one obtains a correction to the decay width:

$$\Gamma_2 = \frac{e^2}{16\pi s^2} \langle \frac{4\pi\alpha}{\bar{q}^2} \rangle F. \quad (2.141)$$

The fact that the gauge dependent terms in the sum $\Gamma_1 + \Gamma_2$ cancel is thus traced back to the identity 2.139. The gauge independent contribution from first order BS-perturbation theory is now different from that of [41] (actually it gives now the net result) in accordance with the different zero order equation, used in our present context.

To complete our calculation we finally check the contributions of second order perturbation theory. It can be shown that due to the fact that the first weak correction now is already of order $O(\alpha_s^2 \Gamma)$, this corrections do not contribute to the required order. Thus our present approach considerably simplifies the calculation of bound state corrections to the decay width.

Summing up all contributions we get the result

$$\Delta\Gamma_{\text{boundstate}} = \Gamma_1 + \Gamma_2 + \Gamma_3 = -\Gamma_0 \frac{\sigma_n^2}{2} \quad (2.142)$$

in agreement with our previous calculation in the last section and [41].

2.5.3 A General Theorem on Bound State Corrections

The above considerations suggest that the result 2.142 is only a special case of a more fundamental statement, with broader range of applicability, which we will now derive [48].

Consider a fermion whose decay can be described by the imaginary part of a self energy function which will have the general form (in any covariant gauge in the relevant sector of the theory)

$$\Sigma(p) = \Sigma_S(p^2) + \not{p}\Sigma_V(p^2) + \gamma_5\Sigma_P(p^2) + \not{p}\gamma_5\Sigma_A(p^2). \quad (2.143)$$

Within the expectation value 2.59 the pseudoscalar and axial vector parts vanish. Furthermore we will drop all factors $\Theta(p^2 - \mu^2)$, which is a valid approximation in all cases where $\alpha^2 m \ll m - \mu$. If this is not the case, either the whole calculation makes no sense because then one would have to include the entire rung of Coulomb interactions of the remaining particle with the heavy decay products or the decay width becomes very small, or both. Let us therefore consider the correction to the decay width which results from the remaining parts of the self energy in an on-shell renormalization scheme (m is the pole mass of the fermion). We expand Σ around that mass shell:

$$\Sigma(p) = i(\text{Im}\Sigma_S + \not{p}\text{Im}\Sigma_V) + (\Sigma'_S + \not{p}\Sigma'_V)(p^2 - m^2). \quad (2.144)$$

Terms of $O(\alpha_{weak}^2)$ are understood to be neglected (cf. remarks after eq. 2.135) and we denote

$$\begin{aligned} \Sigma_X &:= \Sigma_X(m^2), & X = S, V \\ \Sigma'_X &:= \left. \frac{\partial}{\partial p^2} \Sigma_X(p^2) \right|_{p^2=m^2}. \end{aligned}$$

From eqs. 2.139 and 2.144 we can easily calculate the vertex correction, say, for the upper particle line:

$$\Lambda_0 \lambda^+ = g T^a \gamma_0 [i\text{Im}\Sigma_V + 2m(\Sigma'_S + m\Sigma'_V)] \lambda^+ \quad (2.145)$$

After some algebra we can write to the required order

$$\begin{aligned} h_0^{(\Sigma)} &= (\text{Im}\Sigma_S + m\text{Im}\Sigma_V + \frac{\Gamma}{2}\gamma_0) \otimes (\not{p}_2 - m) - iD_{0,\Gamma=0}^{-1} [i\text{Im}\Sigma_V + 2m(\Sigma'_S + m\Sigma'_V)] \\ &\quad (2.146) \end{aligned}$$

$$h_0^{(\Lambda)} = -\frac{4\pi\alpha}{\vec{q}^2} [i\text{Im}\Sigma_V + 2m(\Sigma'_S + m\Sigma'_V)] \gamma_0 \otimes \gamma_0$$

Since

$$\Gamma = -2\bar{u}\text{Im}\Sigma u\Big|_{p=(m,\vec{0})} = -\text{Im}\Sigma_S(m^2) - m\text{Im}\Sigma_V(m^2) \quad (2.147)$$

and $(iD_{0,\Gamma=0}^{-1} - K_{0,\Gamma=0})\chi = 0$ we obtain for the bound state correction to the decay with to $O(\alpha_s^2\alpha_{weak})$

$$\Delta\Gamma_{boundstate} = -4i\Gamma\langle\langle\lambda^- \otimes (\not{p}_2 - m)\rangle\rangle = -\Gamma_0\frac{\langle\vec{p}^2\rangle}{2m^2} \quad (2.148)$$

Now it is clear from the preceding argument that it applies not only to the toponium system but also to all other systems which can be described as weakly bound fermion-antifermion system with unstable components. Another example which is known for a long time is provided by the bound muon system where the above result has also been obtained first by explicit calculation [49]. This result is easily reproduced from 2.148 by replacing Γ_0 by the free muon decay width and m by the muon mass. The effect of the reduced mass (in our case $m/2$, negligible for the bound muon) is entirely included in the expectation value. We conclude that the formalism developed here and especially the use of the identity 2.139 simplifies the problem of the bound state correction to the decay width in a profound way. It is now possible to clearly isolate the underlying cancellation mechanism which automatically gives a gauge independent result which can be interpreted as time dilatation alone. However, because we may now also include the scalar part of the self energy function and since K_0 can be any kernel, the result 2.148 is found to comprise [41] and [49] as special cases of a more general theorem: The leading bound state corrections for weakly bound systems of unstable fermions (with decays like $t \rightarrow b + W^+$, $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$) are *always* of the form 2.148. Among the consequences we especially note that 2.148 may be safely applied to toponium even if one uses e.g. a renormalization group improved potential. While we have presented a field theoretically consistent picture of the "smearing" of the individual levels in toponium below threshold, it must be stressed that the treatment of the entire threshold region cross section also requires an extension of our study to the "continuum" $P_0 > 2m_t$ for which the existing BS formalism must be adapted.

Chapter 3

Heavy Quark Production in e^+e^- Colliders

In the last chapters we calculated the potential and bound state corrections to the decay width with rigorous field theoretical methods. These methods will be necessary to compare theory and high precision measurements. However, for the near future [33] it will probably be sufficient to apply existing less rigorous methods [34]. In the case of quickly decaying particles near threshold the Green function method [35, 44, 45] seems appropriate. Within this approach the four point function is approximated by a nonrelativistic Green function fulfilling an ordinary Schrödinger equation with an appropriate potential. This method works to $O(\alpha_s)$ since the respective corrections to the decay width and potential as well as the hard corrections decouple to this order [51, 52]. At $O(\alpha_s^2)$ this decoupling can be expected to break down and thus it turns out that the calculation of the cross section of decaying particles to order of 1% requires a deeper understanding of the relativistic effects near threshold. A completely satisfying and practically tractable method does not yet exist.

In this chapter we will investigate the production cross section of a $t\bar{t}$ pair near threshold. Whenever possible we will use the results of the foregoing chapters.

The cross section for the process $e^+e^- \rightarrow X$, where X is a set of final states X_n can be written as

$$\sigma = \frac{1}{2K(P^2)} \sum_n |\langle X_n | T | e^+e^- \rangle|^2 \quad (3.1)$$

with

$$K(P^2) = \sqrt{P^2(P^2 - 4m_e^2)}. \quad (3.2)$$

If we neglect initial state interaction and calculate the matrix element to the first order in the electromagnetic coupling we can write for the s-channel:

$$\begin{aligned} M = \langle X|T|e^+e^- \rangle &= (2\pi)^4 \delta(P - t - t') \bar{v}_\alpha(t') \left[e\gamma_\mu \frac{g^{\mu\nu}}{P^2} \langle X|J_\nu^{(\gamma)}(0)|0 \rangle - \right. \\ &\quad \left. - \frac{e}{2sc} \gamma_\mu (v_e - a_e \gamma_5) \frac{g^{\mu\nu}}{P^2 - M_Z^2 + iM_Z \Gamma_Z} \langle X|J_\nu^{(Z)}(0)|0 \rangle \right] u_\beta(t) \end{aligned} \quad (3.3)$$

The SM values for the vector and axial coupling of the Z-boson read

$$v_f = T_f^3 - 2Q_f \sin^2 \theta_w \quad (3.4)$$

$$a_f = T_f^3 \quad (3.5)$$

where T_f^3 is eigenvalue of the third component of the weak isospin operator. For the top quark we have $T_t^3 = 1/2$ and $Q_t = 2/3$.

We are only interested in the contributions of one flavor to the total cross section. Investigating the top quark contribution, we have to replace the currents which in general include a sum over all kinds of particles by the current involving only the top quark field operator $\Psi(x)$. Splitting the currents into vector and axial vector parts we get

$$\begin{aligned} J_\nu^{(\gamma)}(x) &= iq_t e J_\nu(x), & J_\nu^{(Z)} &= \frac{-iev_t}{2sc} J_\nu(x) + \frac{iea_t}{2sc} J_\nu^5(x) \\ J_\nu(x) &= : \bar{\Psi}(x) \gamma_\nu \Psi(x) :, & J_\nu^5(x) &= : \bar{\Psi}(x) \gamma_\nu \gamma_5 \Psi(x) : \end{aligned} \quad (3.6)$$

where from now on we abbreviate

$$s = \sin \theta_w \quad (3.7)$$

$$c = \cos \theta_w$$

Substituting eq. 3.6 into eq. 3.3 we get for the matrix element M

$$\begin{aligned} M &= (2\pi)^4 \delta(P - t - t') \bar{v}_\alpha(t') (-ie^2) \frac{\gamma^\mu}{P^2} \left[(a^{(V)} + b^{(V)} \gamma_5) \langle X|J_\mu(0)|0 \rangle \right. \\ &\quad \left. + (a^{(A)} + b^{(A)} \gamma_5) \langle X|J_\mu^5(0)|0 \rangle \right] u_\beta(t) \end{aligned} \quad (3.8)$$

where

$$a^{(V)} = q_t - \frac{v_t v_e P^2}{4s^2 c^2 z}, \quad b^{(V)} = \frac{v_t a_e P^2}{4s^2 c^2 z}, \quad (3.9)$$

$$a^{(A)} = \frac{a_t v_e P^2}{4s^2 c^2 z}, \quad b^{(A)} = -\frac{a_t a_e P^2}{4s^2 c^2 z}, \quad (3.10)$$

$$z = P^2 - M_Z^2 + iM_Z \Gamma_Z, \quad (3.11)$$

and t, t' denote the momenta of electrons and positrons, respectively. Since the mass of the top quark is much larger than the mass of the Z-boson we can neglect the width of the Z-boson in 3.11. This means that we can treat the $a^{(X)}$ and $b^{(X)}$ as real valued.

For the production of heavy scalars the above formulas can also be used, but only the vector current is present and has to be replaced by

$$J_\mu(x) = i : (\Phi^*(x) \partial_\mu \Phi(x) - \partial_\mu \Phi^*(x) \Phi(x)) : \quad (3.12)$$

where Φ represents the scalar field for a given mass eigenstate. To be definite we will identify this scalar field in the following with the lightest scalar partner of the top quark as predicted by the minimal supersymmetric SM. For a given value θ_t of the L/R mixing angle, the vertices for the lighter particle can be written as $ie\tilde{Q}(p_1^\mu + p_2^\mu)$ with [53]

$$\tilde{Q}_\gamma = -q_t, \quad (3.13)$$

$$\tilde{Q}_Z = (\cos^2 \theta_t - 2q_t \sin^2 \theta_W)/(2sc). \quad (3.14)$$

The coefficients $a^{(V)}$ and $b^{(V)}$ have thus to be replaced by

$$a_s = -q_t + \frac{\tilde{Q}_Z P^2}{2scz}, \quad b_s = -\frac{\tilde{Q}_Z a_e P^2}{2scz}, \quad (3.15)$$

respectively. The lepton trace for positrons with polarization ρ' and electrons with polarization ρ ($m_e \rightarrow 0$) can be readily calculated to yield

$$\begin{aligned} L^{\mu\nu} &= \frac{1}{4} Sp[(1 - \rho' \gamma_5) \gamma^\sigma \gamma^\mu (a^{(X)} + b^{(X)} \gamma_5) (1 + \rho \gamma_5) \gamma^{\bar{t}} \gamma^\nu (a^{(Y)} + b^{(Y)} \gamma_5)] t'_\sigma t_{\bar{t}} = \\ &= [(a^{(X)} a^{(Y)} + b^{(X)} b^{(Y)}) (1 - \rho \rho') + (a^{(X)} b^{(Y)} + b^{(X)} a^{(Y)}) (\rho - \rho')] (t'^\mu t^\nu + t^\mu t'^\nu - t'^\sigma t_\sigma g^{\mu\nu}) - \\ &\quad - i[(a^{(X)} b^{(Y)} + b^{(X)} a^{(Y)}) (1 - \rho \rho') + (a^{(X)} a^{(Y)} + b^{(X)} b^{(Y)}) (\rho - \rho')] \epsilon^{\alpha\mu\beta\nu} t'_\alpha t_\beta \end{aligned} \quad (3.16)$$

This simplifies in the CM frame to a purely space like tensor. Let us now focus on the total cross section. Since P^i is zero in the CM frame, the hadronic tensor T_{ik} has to be proportional to δ_{ik} and therefore the antisymmetric part in L vanishes in the calculation

of the total cross section. However, this part of the lepton trace will lead to a forward backward asymmetry as discussed below. It is reasonable to expect that a future linear e^+e^- collider will work with polarized electrons and unpolarized positrons. Therefore we will assume this scenario in the following. The generalization to the general case is obvious from 3.16.

For unpolarized positrons ($\rho' = 0$) the symmetrical contraction of L simplifies to

$$L^{ik}\delta_{ik} = [a^{(X)}a^{(Y)} + b^{(X)}b^{(Y)} + (a^{(X)}b^{(Y)} + b^{(X)}a^{(Y)})\rho]P^2 \quad (3.17)$$

The modulus squared of the hadronic matrix elements can be simplified [29] according to

$$\sum_X (2\pi)^4 \delta(P - P_X) \langle 0 | j_\mu^A(0) | X \rangle \langle X | j_\nu^B(0) | 0 \rangle = 2 \text{abs} T_{\mu\nu}^{AB} \quad (3.18)$$

where

$$\begin{aligned} T_{\mu\nu}^{XY} &= i \int d^4x e^{iPx} \langle 0 | T j_\mu^X(x) j_\nu^Y(0) | 0 \rangle = \int \frac{d^4p}{(2\pi)^4} \frac{d^4p'}{(2\pi)^4} Sp[\Gamma_\mu^X G(P, p, p') \Gamma_\nu^Y] = \\ &=: \left(\frac{P_\mu P_\nu}{P^2} - g_{\mu\nu} \right) G^{XY}(P) \end{aligned} \quad (3.19)$$

and the indices X, Y refer to the vector and axial currents as defined in 3.6 and 3.12. Here we made explicit the connection to the four point function $G(P, p, p')$ defined in 2.5. Due to parity conservation the mixed V-A currents vanish to all orders in the strong coupling constant. The electroweak interaction may well introduce parity non-conserving effects but near threshold the heavy particles should be almost on shell and thus such effects should be suppressed at least by one order in the electroweak coupling for the total cross section. Thus we can split off the total cross section into a purely vector and axial-vector contribution.

$$\sigma = \sigma_V + \sigma_A \quad (3.20)$$

For fermions the vector part is the dominant one near threshold, since it allows the production of S-waves (with spin $S = 1$). The axial contribution is suppressed by two powers in α_s .

For scalars we have only the vector contribution, but in the nonrelativistic limit the coupling proportional to spatial components p^i (in the CM frame) selects $l = 1$ states. Thus the production of scalars is also suppressed by two powers in α_s .

Collecting all ingredients from above and multiplying by a color factor $N = 3$ we have for the contribution to the total cross section from the flavor under consideration :

$$\sigma_X = \frac{72\pi}{P^2} [c_X + d_X \rho] \text{Im} G^{XX}(P) \sigma_{\mu\bar{\mu}} \quad (3.21)$$

$$\sigma_s = \frac{72\pi}{P^2} [c_s + d_s \rho] \text{Im} G_s(P) \sigma_{\mu\bar{\mu}} \quad (3.22)$$

with

$$\begin{aligned} c_X &:= (a^{(X)})^2 + (b^{(X)})^2 & c_s &= a_s^2 + b_s^2 \\ d_X &:= 2a^{(X)}b^{(X)} & d_s &= 2a_s b_s \end{aligned} \quad (3.23)$$

and

$$\sigma_{\mu\bar{\mu}} = \frac{4\pi\alpha_{QED}^2(m_t)}{3P^2} \quad (3.24)$$

3.1 Top - Antitop Production Cross Sections Near Threshold

3.1.1 Total Cross Section

For the calculation of the total cross section for $t\bar{t}$ production in the threshold region the large width of the toponium system plays a crucial role. It has the disadvantage that individual levels disappear since the width becomes comparable to the $1s - 2s$ splitting. While corrections to each pole, which lie now in the unphysical sheet, as described above, are in principle possible, such a calculation becomes untractable in practice. However, as already mentioned in the introduction, the large width provides an effective cut-off for nonperturbative corrections and renormalons. This will be demonstrated explicitly in the following. To circumvent the calculation of corrections separately for each level and afterwards summing those to obtain the Green function, it is convenient to calculate it numerically and use eq. 3.21 for the total cross section.

As a first approximation in the region near threshold it should be possible to replace the exact Green function by the nonrelativistic one. This Green function fulfills the Schrödinger equation

$$\left\{ -\frac{\Delta}{m} + V(r) - E - i\Gamma \right\} \tilde{G}(\vec{r}, \vec{r}') = -\delta(\vec{r} - \vec{r}') \quad (3.25)$$

For the potential $V(r)$ we would like to use the potential 2.108 since it is e.g. a systematic expansion in the strong coupling constant and thus is expected to lead to gauge independent results. However, the inclusion of higher order corrections will lead to some problems, as we will see below.

The leading contributions to 3.20 arises from the vector coupling (e.g. $X = V$ in 3.21) since it produces $t\bar{t}$ pairs with angular momentum zero (S-waves). Therefore we are interested in $l = 0$ solutions of eq. 3.25. Due to the symmetry of the Green function and the requirement of regularity at $r \rightarrow \infty$ and at the origin, the general solution of eq. 3.25 can be written as

$$\tilde{G}(\vec{r}, \vec{r}') = \sum_{l=0}^{\infty} g_l(r, r') \sum_{m=-l}^l Y_{lm}^*(\Omega') Y_{lm}(\Omega) \quad (3.26)$$

$$g_l(r, r') = \frac{g_{<}(r_{<})g_{>}(r_{>})}{r_{<}r_{>}}, \quad r_{>} = \begin{cases} r : r > r' \\ r' : r' > r \end{cases} \quad r_{<} = \begin{cases} r : r < r' \\ r' : r' < r \end{cases} \quad (3.27)$$

where $g_{<}(r_{<})$ and $g_{>}(r_{>})$ are regular solutions of the homogeneous equation

$$\left\{ \frac{\partial^2}{\partial r^2} - m(E + i\Gamma - V(r)) - \frac{l(l+1)}{r^2} \right\} g(r) = 0 \quad (3.28)$$

at $r = 0$ or $r \rightarrow \infty$, respectively. The actual behavior of the solution $g_{<}(r_{<})$ (and also of the irregular one) for $r \rightarrow 0$ depends on the potential and on l . Due to the presence of the width $i\Gamma$ the behavior of $g_{>}(r)$ for $r \rightarrow \infty$ is given by

$$\lim_{r \rightarrow \infty} g_{>}(r) \propto e^{-a_- r} \quad (3.29)$$

with

$$a_- := \sqrt{\frac{m}{2}} \sqrt{-E + \sqrt{E^2 + \Gamma^2}}. \quad (3.30)$$

This exponential damping behavior is the origin of the infrared cut off mentioned already in [35]. Consider for simplicity the case $E = 0$. Then we can write

$$a_-^{-1} = \sqrt{\frac{2}{m\Gamma}} = \sqrt{\frac{E_B}{\Gamma}} r_B \quad (3.31)$$

where $E_B = m\alpha^2/2$ and r_B are the Bohr energy and the Bohr radius of the system, respectively. This formula clarifies that if the width Γ is approximately equal to E_B the

form of the potential is only "tested" up to the order of magnitude of the Bohr radius. The situation is improved for $E < -\Gamma$ since here we have

$$a_-^{-1} \leq \sqrt{\frac{1}{m|E|}}, \quad E < -\Gamma.$$

This relation also holds also for small Γ and thus the Bohr radius again becomes the relevant scale.

The matching condition for $g_<$ and $g_>$ is provided by the δ distribution in eq. 3.25:

$$-m = g_<(r)g'_>(r) - g'_<(r)g_>(r) \quad (3.32)$$

Numerically it is possible to obtain the regular solution at the origin directly by imposing suitable boundary conditions. For the singular solution we use the following method [44, 45]. Suppose you have two solutions: $g_<(r)$ as above and $u_2(r)$ an solution determined by an arbitrary boundary condition. Then the solution $g_>$ is given by

$$g_>(r) = c[u_2(r) + Bg_<(r)] \quad (3.33)$$

with

$$B = -\lim_{r \rightarrow \infty} \frac{u_2(r)}{g_<(r)}. \quad (3.34)$$

The constant c can be determined from eq. 3.32. For $l = 0$ and a Coulomb-like potential at the origin we arrive with the boundary conditions

$$\begin{aligned} g_<(0) &= 0 \\ g'_<(0) &= 1 \end{aligned} \quad (3.35)$$

and arbitrary ones for u_2 at the result

$$\tilde{G}_{l=0}(\vec{r}, 0) = \frac{m}{4\pi w} \frac{u_2(r) + Bg_<(r)}{r}, \quad (3.36)$$

$$w = u_2 g'_< - g_< u'_2. \quad (3.37)$$

According to eq. 3.21 the total cross section is proportional to $\text{Im}\tilde{G}(0,0)$. With this knowledge it is possible to calculate the total cross section near threshold for any given potential $V(r)$. We have written a numerical routine for "MATHEMATICATM" which has been checked to give the correct answer for the known Green function of a purely Coulombic potential.

Before presenting some numerical studies, a short remark on the coupling constant to be used seems in order. It is customary to take the strong coupling constant α_s in the $\overline{\text{MS}}$ scheme. Especially experimental determinations are always given in terms of $\alpha_s^{\overline{\text{MS}}}(M_Z)$. But in bound state calculations it is natural to use an α_s defined differently as we did in the foregoing. Clearly it should be possible to relate the two schemes. To avoid complications from heavy fermion masses we treat 4 flavors as massless. The difference in the bottom quark contribution can be expected to be smaller than the experimental uncertainties in α_s . Therefore we ignore it in the following consideration. The quark-antiquark potential in the $\overline{\text{MS}}$ scheme can be extracted e.g. from [54] to $O(\alpha_s)$.

$$\mathcal{V} = -\frac{4\pi\alpha}{\vec{q}^2} \left[1 - \alpha(3\pi\beta_0 \ln \frac{\vec{q}^2}{\mu^2} + \frac{31}{16\pi} - \frac{10n_f}{16\pi}) \right] \quad (3.38)$$

Comparison with 2.76 for n_f light flavors gives

$$\alpha^{BS} = \alpha_{\overline{\text{MS}}} \left(1 - \frac{\alpha_{\overline{\text{MS}}}}{16\pi} (31 - \frac{10n_f}{3}) \right) \quad (3.39)$$

where BS denotes our bound state scheme. Numerically this means that our α is slightly smaller than usual which is advantageous for our perturbative calculation. For $m_t = 180$, $\alpha_s^{\overline{\text{MS}}}(M_Z) = .117 \pm 0.05$ get for a renormalization at $\mu = 1/r_B(\mu)$

$$\mu = 18.5 \pm 1 \text{ GeV} \quad (3.40)$$

$$\alpha^{\overline{\text{MS}}}(\mu) = 0.22 \pm 0.01 \quad (3.41)$$

$$\alpha(\mu) = 0.19 \pm 0.01 \quad (3.42)$$

Remember that the above values for $\alpha, \alpha^{\overline{\text{MS}}}$ differ by a factor $4/3$ from α_s .

For the comparison of the two schemes the terms of the form α_s^n/\vec{q}^2 were important. To compare to relative $O(\alpha_s^2)$ terms up to α_s^3/\vec{q}^2 are needed in the potential. While the only term of this form in our scheme has been calculated in our present work (eq. 2.90), we are not aware of a calculation of the terms of the order $\text{const}\alpha_s^2/\vec{q}^2$ in the $\overline{\text{MS}}$ scheme. Therefore, it is at present possible to compare the strong coupling constant in these two schemes only to $O(\alpha_s)$.

Let us first compare fixed order perturbation theory with the renormalization group improved one. This corresponds to an investigation of the effect of the gluon loops given in 2.76 and 2.85. The leading term is of the form $(\gamma + \ln \mu r)$. This term will appear in any higher order. Therefore we add to the pure Coulomb exchange successively terms of the form $(\gamma + \ln \mu r)^n$ coming from the leading log of the n -gluon loop.

The results are shown in fig.3.1. The uppermost line corresponds to a pure Coulomb potential while the line slightly below has been calculated including the contribution of the bottom quark. For all other curves the length of the segments is a measure for the number of logarithmic terms (e.g. from top to bottom: $n = 2, 4, \infty, 5, 3, 1$).

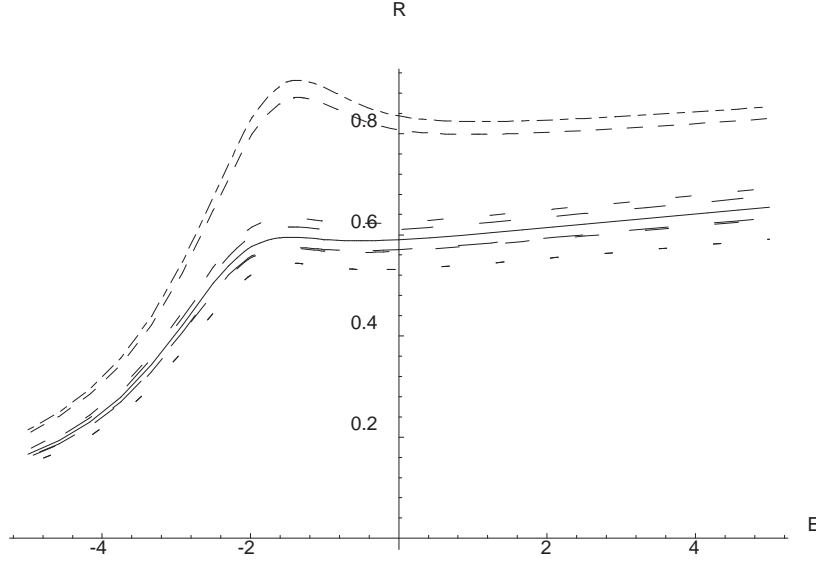


Fig. 3.1

Thus we conclude that the inclusion of the effects of the gluon loops is important. Therefore the strict application of perturbation theory is not very useful. Instead one is forced to sum all leading logs from the gluon loop corrections to the Coulomb line. Here one encounters the problem that the resummed gluon propagator has a nonintegrable pole and its Fourier transformation is thus not well defined.

We will impose the following recipe to obtain a resummed potential¹. We calculate the corrections perturbatively in momentum space, perform the Fourier transformation for each term and then sum up all terms proportional to $(\gamma + \log \mu r)$. This leads to a geometrical series which serves as the definition for the potential for values of r where the sum does not converge.

The pole of the geometric series lies at

$$r_{pole} = \frac{1}{\mu} e^{\frac{2\pi}{(11-2/3n_f)\alpha_s} - \gamma}. \quad (3.43)$$

¹Another possibility frequently used in the literature is to use the two loop renormalization group improved potential

We expect any regularisation which removes this apparently unphysical pole to influence the potential at distances of that order of magnitude. For a top quark of about 180 GeV we find

$$r_{pole} \approx \frac{100}{\mu} \quad (3.44)$$

which is about two orders of magnitude larger than the inverse damping constant for this mass (eq. 3.31) if the renormalization scale is chosen as

$$\mu = \frac{1}{r_B} \quad (3.45)$$

or smaller, which is also suggested by the momentum scale relevant for the bound state problem. Therefore, we can safely sum the gluon loops and we will see that the determination of the cross section is only sensitive to quantities well below the pole.

Since the resulting potential is also the solution of the one-loop renormalization group equation for massless loop quarks it is also one loop renormalization scale independent. The contributions from the nonleading log's in 2.85 and from the mass of the c, b quarks are small and can thus be treated in a perturbative manner. Since we have taken only the leading logs from a definite set of diagrams it is clear which contribution has already been taken into account and which one not.

Let us first briefly compare the numerical approach with the standard bound state approach where individual bound states are considered and corrections are calculated in a systematic way to the position of the pole and to the wave functions as described in sect. 2.3.

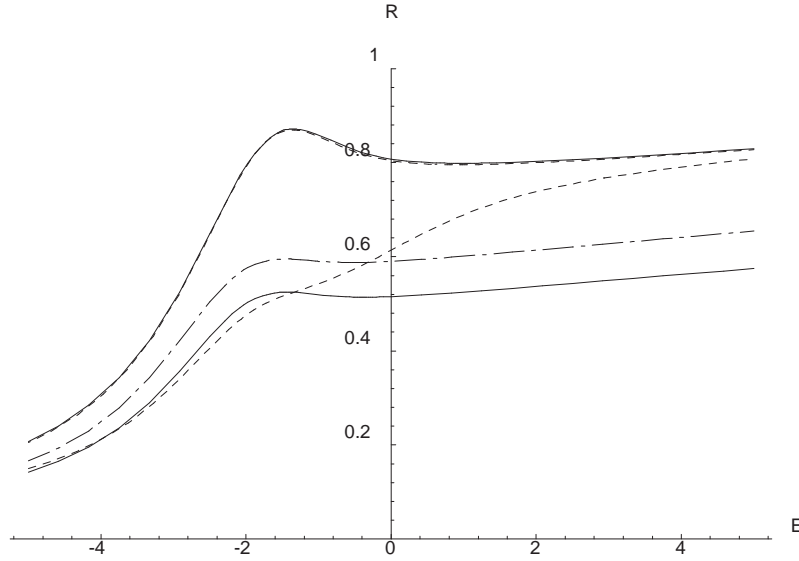


Fig. 3.2

In fig. 3.2 the 1s peak is corrected with the $O(\alpha_s)$ corrections for the position of the pole and the corrections of the wave functions [12] (without correction to the continuum wave functions). Solid lines are calculated within the summation approach, dashed lines with the Green function approach for $m_t = 180$ and an electron polarisation of 0.6. The upper curves correspond to a pure Coulomb potential. The lower curves are from a potential including only the one loop correction. The dashed-dotted curve is the renormalisation group improved result.

The mass shift for the levels can be obtained from the relevant part of the potential by performing the Fourier transformation into coordinate space, where the integrations can be done analytically [28]. The surprisingly simple result is

$$\Delta M_g = \langle H_g \rangle = -m\alpha^3 \frac{11N}{16\pi n^2} [\Psi_1(n+l+1) + \gamma + \ln \frac{\mu n}{\alpha m}] + O(\alpha^5) \quad (3.46)$$

where Ψ_n is the n-th logarithmic derivation of the gamma function and γ denotes Euler's constant.

The picture shows that this approach would give a good result for the position of the rising edge of the cross section as well as for the position of the 1s peak if it were not necessary to sum up the leading logs.

In view of the above mentioned complications we will always include the resummed potential in the following.

Let us now return to the bound state correction for the toponium width.

As can be seen by inspection of eq. 2.148 we could have obtained the correct results for the level shifts simply by calculating the expectation value 2.148 between a wave function of a Schrödinger equation with a QCD potential. The term

$$H_1 = i\Gamma \frac{\vec{p}^2}{2m^2} \quad (3.47)$$

can thus be viewed as an absorptive part of the Hamiltonian. Adding this term is equivalent to the introduction of a "momentum dependent width" [45, 47]. It has been shown [45, 51] that the $O(\alpha_s)$ corrections to the absorptive part of the Hamiltonian vanish.

Following this philosophy to estimate the effect of the bound state corrections to the width we use the Hamiltonian

$$H = \frac{\vec{p}^2}{m} + i\Gamma \frac{\vec{p}^2}{2m^2} + V_{QCD}(r) \quad (3.48)$$

where V_{QCD} is the one loop resummed potential.

The results of our numerical studies for $m_t = 180, \rho = 0$ are shown in fig.3.3 and 3.4.

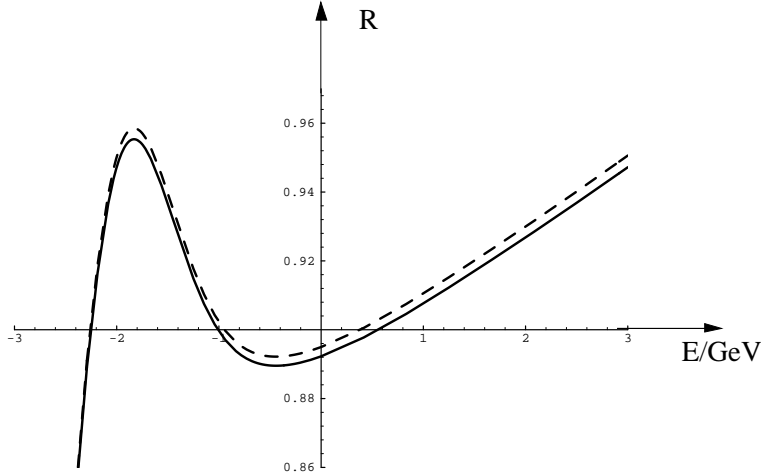


Fig. 3.3: Effect of the boundstate corrections to the toponium decay width on the total cross section near threshold in units of $\sigma_{\mu\bar{\mu}}$. Full line: without, dashed line with bound state corrections to the decay width.

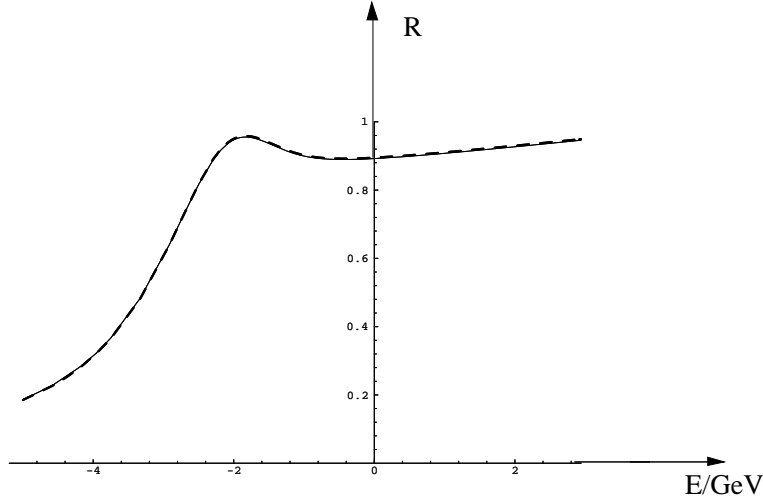


Fig. 3.4: The same as Fig. 3.3 with a different scaling - demonstrating the smallness of the effect.

The cross section is slightly enhanced and the effect is of the order of less than 1% and thus "really" an $O(\alpha_s^2)$ effect. However, we would like to indicate that this enhancement is not only at the position of the 1S resonance as it would be expected from a simple summation over resonances and from the model calculation in ref. [47]. Furthermore, it is interesting to note that the Hamiltonian 3.48 gives the same result as a similar one with the second term missing but with the replacement $m \rightarrow m - i\Gamma/2$. This convinces us that H_1 can be considered as small.

While the above result in the Green function approach shows reasonable results, we nevertheless want to add some words of caution. One observation concerns the definition of the perturbed resolvent. Assuming that relativistic effects are of the same order of magnitude and can be included in a perturbative manner, it seems necessary to go back to the perturbative expansion and subtract the already included parts. But due to our nonrelativistic approximation $H_1 \propto \vec{p}^2$ the second term in the series

$$\text{Im}G = \text{Im}G_0 + \text{Im}(G_0 H_1 G_0) + \dots$$

is divergent while the relativistic expression is clearly well behaved. Also the cancellation of gauge dependent contributions coming from the high momentum region seems to be more involved than it is for the level corrections.

The second observation concerns the comparison of the results of fig 3.3 with an approach where one calculates corrections to the wave functions and to the position of the poles and afterwards takes the sum over the whole spectrum to obtain the Green function

[46]. This, in general, cannot lead to a correct result since H_1 makes the Hamiltonian non hermitian and thus the latter need not have a spectral representation.

We conclude that certain electroweak and strong corrections of the same order of magnitude must be still missing or, even worse, it is not yet clear how to correctly add them. Thus a better theoretical understanding of such effects is desirable.

However, these effects will be of the same order of magnitude and we thus may conjecture that the theoretical uncertainties of the calculation of the $t\bar{t}$ cross section near threshold remain at the percent level.

An effect larger than that could be induced by a relatively light Higgs boson. This assumption is in agreement with recent electroweak data and the corresponding fit to the MSSM. If the MSSM and the Higgs mechanism turn out to be realized in nature the lightest higgs has to lie in the range 90 to 130 GeV. Recent numerical studies [55] show an enhancement of the cross section at the 10% level for $m_H \approx 100\text{GeV}$ which seems to be the value presently favoured by the analysis of radiative corrections to the SM and experimental limits.

3.1.2 Forward-Backward Asymmetry

In the last subsection we investigated the total cross section for $t\bar{t}$ production in a future e^+e^- collider. This should be rather good observable to determine the mass of the top quark. But since the total cross section at threshold is also significantly determined by other quantities like the strong coupling constant and the top decay-width, it is necessary to have some independent observables. It was first proposed in [56] that the forward backward (FB) asymmetry would provide such a quantity. It has the additional advantage that it is independent of the absolute normalization of the cross section measurement and is therefore a quantity which can be determined experimentally with high accuracy. However, treating this problem in a nonrelativistic context, one encounters unphysical divergences which made necessary the introduction of an unnatural cut-off. In this subsection we will show a systematic way how to avoid such divergencies.

The FB asymmetry is defined as the relative difference of the cross section for particles produced in the forward and in the backward direction, with respect to the direction of the e^- beam.

$$A_{FB} := \frac{\sigma_{FB}}{\sigma_{tot}} = \frac{\int_0^{\frac{\pi}{2}} d\theta \frac{d\sigma}{d\theta} - \int_{\frac{\pi}{2}}^{\pi} d\theta \frac{d\sigma}{d\theta}}{\sigma_{tot}} \quad (3.49)$$

Since the top quarks will be identified by their main decay products W and b let us consider the cross section for the process $e^+e^- \rightarrow t\bar{t} \rightarrow bW^+\bar{b}W^-$. Production of $W^+bW^-\bar{b}$ via other channels can be treated as in [60].

$$d\sigma_{e^+e^- \rightarrow bW^+\bar{b}W^-} = \frac{1}{2P^2} |M_A + M_V|^2 d\Phi_{W+b} d\Phi_{W-\bar{b}} (2\pi)^4 \delta(t_1 + t_2 - k_1 - k_2 - b_1 - b_2) \quad (3.50)$$

with

$$d\Phi_{W+b} = \frac{d^4b}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} (2\pi) \Theta(k_{10}) \delta(k^2 - M^2) (2\pi) \Theta(b_{10}) \delta(b_1^2) \quad (3.51)$$

and similarly for the decay of the antitop.

The FB asymmetry originates in the interference terms of the vector coupling (M_V) and the axial coupling (M_A). This is depicted in figure 3.5 where V indicates a vector and A an axial vector coupling, the star denotes complex conjugation. The matrix element M is essentially the vertex $\gamma t\bar{t}$ and $Z t\bar{t}$. Near threshold the perturbative treatment of the Coulomb interaction is no longer valid. Instead one has to include the whole rung of

$$\begin{aligned}
d\sigma_{\text{FB}} = & \left(\text{wavy line with } \mathbf{V} \text{ and } \mathbf{A} \text{ vertices} \right) * + \left(\text{wavy line with } \mathbf{V} \text{ and } \mathbf{A} \text{ vertices and a loop} \right) * + \\
& + \left(\text{wavy line with } \mathbf{V} \text{ and } \mathbf{A} \text{ vertices and a loop with a dashed line} \right) * + \dots
\end{aligned}$$

Fig. 3.5

Coulomb interactions and even worse, it is necessary to include the leading logarithmic contributions from the gluonic self energy corrections, as we have seen in the last section. We can sum the relevant set of graphs by means of the equation

$$\Gamma^\mu = \gamma^\mu - iKD\Gamma^\mu \quad (3.52)$$

for the vertex function Γ^μ . Comparison with the BS-Equation 2.26 shows that it has the solution

$$\Gamma_V^\mu = -iD^{-1}G\gamma^\mu \quad (3.53)$$

$$\Gamma_A^\mu = -iD^{-1}G\gamma_5\gamma^\mu, \quad (3.54)$$

where it is understood, that the two legs of G on the right hand side are connected with the γ -matrices and a momentum integration is performed.

We can split the matrix element into

$$M_A^i = m_1 m_2 D\Gamma_A^i \quad (3.55)$$

$$M_V^i = m_1 m_2 D\Gamma_V^i$$

where

$$m_1 = \epsilon_\mu^{(\sigma)}(k_1) \bar{u}_\lambda(b_1) \left(-i \frac{e}{\sqrt{2} \sin \theta_W} V_{33}^* \gamma^\mu P_- \right) \quad (3.56)$$

$$m_2 = \epsilon_\nu^{*(\sigma')}(k_2) \left(-i \frac{e}{\sqrt{2} \sin \theta_W} V_{33} \gamma^\nu P_- \right) v_{\lambda'}(b_2). \quad (3.57)$$

Near threshold the propagators in D and the Green function G should be dominated by small momenta and therefore it should be possible to replace them by their nonrelativistic approximations. However, for D the approximation to $O(\alpha)$ is needed for the axial vertex, since the zero order propagators would give zero due to $\lambda^- \gamma_5 \gamma^i \lambda^+ = 0$.

$$D \rightarrow S_{nr}^+ \otimes S_{nr}^- \quad (3.58)$$

$$S_{nr}^\pm = (\lambda^\pm - \frac{\vec{p}\vec{\gamma}}{2m}) \frac{1}{\frac{E}{2} \pm p_0 - \frac{\vec{p}^2}{2m} + i\frac{\Gamma}{2}} \quad (3.59)$$

This gives rise to the effective nonrelativistic axial vertex

$$\gamma_5 \gamma^i \rightarrow \frac{p^k}{2m} [\gamma^i, \gamma^k] \gamma_5 \lambda^-. \quad (3.60)$$

We note already here that the term $\frac{\vec{p}\vec{\gamma}}{2m}$ introduces an additional power in \vec{p} which is not present in the relativistic propagator. Instead the factor $\frac{p^k}{2m}$ in 3.60 would be replaced by $\frac{p^k}{2E_p}$ which is finite for $p \rightarrow \infty$.

Using the fact that the heavy quarks are on shell up to $O(\alpha^2)$ we can write

$$\int d\Phi_{W+b} \sum_{\sigma, \lambda} |m_1|^2 = -2\text{Im}\Sigma(m^2) \lambda^+ + O(\alpha^2) \quad (3.61)$$

in agreement with section 2.5.

From the modulus squared of the propagators we obtain a factor

$$\int \frac{dp_0}{2\pi} \left| \frac{1}{p_0^2 - \omega^2} \right|^2 = \frac{2}{\Gamma |2\omega|^2}, \quad (3.62)$$

$$\omega = \frac{1}{2m} (\vec{p}^2 - mE - im\Gamma) \quad (3.63)$$

Collecting everything from above, performing the trace in a frame where the leptons move along the z-axis, and using eq. 3.16 we get for the FB-asymmetry

$$\sigma_{FB, \Lambda} = (c_{AV} + d_{AV} \rho) \frac{18\Gamma}{\pi^2 P^2} \int_0^\Lambda p^2 dp d\Omega_{FB} \text{Re}[G^*(\vec{p}) \int \frac{d^3 q}{(2\pi)^3} G(\vec{p}, \vec{q}) \frac{q^{(3)}}{m}] \sigma_{\mu\bar{\mu}} \quad (3.64)$$

where

$$c_{AV} = a^{(A)} b^{(V)} + a^{(V)} b^{(A)} \quad (3.65)$$

$$d_{AV} = a^{(A)} a^{(V)} + b^{(A)} b^{(V)} \quad (3.66)$$

$$\int d\Omega_{FB} = \int_0^{2\pi} d\varphi \left(\int_0^{\frac{\pi}{2}} d\theta - \int_{\frac{\pi}{2}}^\pi d\theta \right) \sin \theta \quad (3.67)$$

The SM values for $a^{(X)}, b^{(X)}$ are given in 3.9 and 3.10. We introduced an intermediate cut-off Λ in 3.64 since the p integration is logarithmically divergent. It will be removed in the following. The Green function $G(\vec{p})$ in 3.64 is defined by

$$G^*(\vec{p}) = \int \frac{d^3q}{(2\pi)^3} G^*(\vec{p}, \vec{q}) = \frac{4\pi}{p} \int_0^\infty dr r \sin pr \tilde{G}_{l=0}(r, 0). \quad (3.68)$$

$\tilde{G}_{l=0}(r, r')$ denotes the S-wave Green function in configuration space, eq. 3.36. Let us now investigate the term

$$\int \frac{d^3q}{(2\pi)^3} G(\vec{p}, \vec{q}) \frac{q^{(3)}}{m} = \frac{-i}{m} \int d^3x e^{-i\vec{p}\vec{x}} \frac{\partial}{\partial y_3} \tilde{G}(\vec{x}, \vec{y})|_{\vec{y}=0} \quad (3.69)$$

Using the representation 3.26 one can show that only P-waves ($l = 1, m = 0$) contribute to the sum. Thus we need to evaluate eq. 3.28 for $l = 1$. The regular and singular solutions behave at the origin as

$$\begin{aligned} g_{<}^{l=1}(r) &\rightarrow c_{<} r^2, \\ g_{>}^{l=1}(r) &\rightarrow c_{>} \frac{1}{r}, \end{aligned} \quad (3.70)$$

respectively. We now construct a singular solution with $c_{>} = 1$ out of two arbitrary solutions u_{p1} and u_{p2} :

$$g_{>}^{l=1}(r) = a_p [u_{p2}(r) - B_p u_{p1}(r)] \quad (3.71)$$

by means of

$$a_p := \lim_{r \rightarrow 0} r^{-1} [u_{p2}(r) - B_p u_{p1}(r)]^{-1}. \quad (3.72)$$

B_p is determined by the requirement of regularity at infinity

$$B_p = \lim_{r \rightarrow \infty} \frac{u_{p2}(r)}{u_{p1}(r)}. \quad (3.73)$$

Then the condition 3.32 demands $c_{<} = m/3$. Performing the differentiation in the z -direction in the limit $y \rightarrow 0$ in 3.69 leads to

$$\int \frac{d^3q}{(2\pi)^3} G(\vec{p}, \vec{q}) \frac{q^{(3)}}{m} = \frac{\cos \theta}{p^2} \int_0^\infty dr \left(\frac{\sin pr}{r} - p \cos pr \right) g_{>}^{l=1}(r). \quad (3.74)$$

While the expressions 3.68 and 3.74 are well defined, it has been observed in [56] and [57] that 3.64 is logarithmically divergent. Thus cutoffs (Λ) have been introduced to

make numerical predictions possible. Since in our present case the divergence is only logarithmical, this leads to phenomenologically reasonable results. But the explanations given in these references are different and so are the cut-offs. It may, however, happen that for the comparison with experimental data with some cuts a reintroduction of some kind of cut-off will be necessary ², but we feel safer by first giving a satisfactory theoretical prediction and leaving this possibility open to the experimentalists.

In any case, the introduction of these cut-offs leads to some numerical uncertainty in the result, and is clearly unsatisfactory from the theoretical point of view. Furthermore we will see in the next section, that a similar divergence occurs in the context of the stop-anti-stop production. Since this divergence is linear, a better understanding of the origin of this kind of divergencies is necessary to give a quantitative prediction.

The key point of our solution of the problem is to go back to the perturbative sum of the graphs for σ_{FB} . This is illustrated in figure 3.5.

Consider now the leading (tree) contribution in the nonrelativistic approximation (e.g. the first graph in fig. 3.5). In this case both S- and P-wave Green functions are replaced by

$$G(\vec{p}, \vec{p}') = \frac{(2\pi)^3 \delta(\vec{p} - \vec{p}')}{\frac{\vec{p}^2}{m} - E - i\Gamma}. \quad (3.75)$$

This leads to the logarithmically divergent expression (for $\Lambda \rightarrow \infty$)

$$\sigma_{FB,\Lambda}^{(1),nr} = \frac{9\Gamma}{2m^3} \int d\Omega_{FB} \int_0^\Lambda dp \frac{p^3 \cos \theta}{|\frac{\vec{p}^2}{m} - E - i\Gamma|^2}. \quad (3.76)$$

Since the remaining graphs in fig. 3.5 by power counting can be shown to give finite results we conclude that the nonrelativistic approximation was not valid in the tree graph due to the extra power in p from the axial vertex. Furthermore since 3.76 is the only divergent contribution in the (infinite, but assumed to be convergent) sum 3.5 representing G we conclude that the divergent part of 3.64 is entirely contained in the first (free) contribution 3.76. We could now remove this divergence by a replacement $p/m \rightarrow p/E_p$, as indicated above. This, however, does not respect another qualitative difference between the exact tree contribution and its nonrelativistic approximation. Namely in the relativistic calculation the phase space is cut off when the momentum squared of one quark falls below the invariant mass of the decay products [45].

Therefore, we return, instead, to the relativistic expression for the tree contribution. Since we are dealing with a decaying particle, we will have to include the self energy

²The author is grateful to M.Jezabek for this information

contribution due to the decay in the propagator. This will in general lead to gauge dependent results, but we can use the constant on-shell width to obtain the leading contribution in the weak coupling [58].

This makes the quantities we are considering finite (for $\Gamma \rightarrow 0$) and gauge independent and the remaining higher order contributions (e.g. gauge dependent if one considers only one diagram) can be calculated perturbatively. Therefore to leading order it seems reasonable to take the constant width approximation for the top propagator.

Since we are only investigating processes with $t\bar{t}$ intermediate states which will give the main contribution to the cross sections we focus on the graph shown in fig. 3.6.

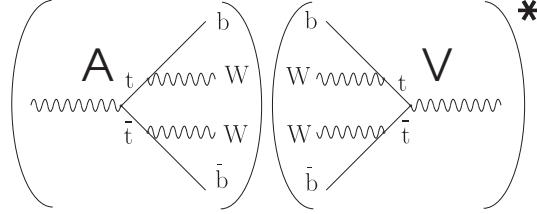


Fig. 3.6

A further advantage of this method is that now the other - non resonant - graphs for the process $e^+e^- \rightarrow W^+bW^-\bar{b}$ to leading order in the weak coupling [60] need only to be added to yield the background contribution.

Performing the straightforward calculation for the relativistic tree contribution with a constant, but non-zero width Γ , we arrive at

$$\sigma_{FB}^{(1)} = \frac{18}{P^2} (c_{AV} + c_{AV}\rho) \int d\mu_1^2 \int d\mu_2^2 \Delta(\mu_1^2) \Delta(\mu_2^2) \left(1 - \frac{(\mu_1 + \mu_2)^2}{P^2}\right) \left(1 - \frac{(\mu_1 - \mu_2)^2}{P^2}\right) \quad (3.77)$$

where

$$\Delta(p^2) = \frac{m\Gamma}{\pi[(p^2 - m^2)^2 + m^2\Gamma^2]} \quad (3.78)$$

This could have been also obtained by replacing the fermion propagator by

$$S = \int d\mu^2 \frac{\Gamma}{\pi[(m^2 - \mu^2)^2 + m^2\Gamma^2]} \frac{(\not{p} + \mu)}{p^2 - \mu^2 + i\epsilon}. \quad (3.79)$$

and cutting it to effectively replace the phase space element for a stable quark

$$d\Phi_{stable} = \frac{d^4p}{(2\pi)^3} \Theta(p_0) \delta(p^2 - m^2)$$

by

$$d\Phi_{unstable} = \frac{d^4p}{(2\pi)^3} \Theta(p_0) \Delta(p^2). \quad (3.80)$$

To conclude we can say that our simple prescription to obtain finite, gauge independent results to the desired order is to replace the nonrelativistic tree contribution by the relativistic one and leave the (finite) rest unchanged:

$$\sigma_{FB} = \sigma_{FB}^{(1)} + \lim_{\Lambda \rightarrow \infty} (\sigma_{FB,\Lambda} - \sigma_{FB,\Lambda}^{(1),nr}) \quad (3.81)$$

In fig. 3.7 the numerical results are shown for the different contributions to σ_{FB} for $\Lambda = 300$ GeV. This can be estimated to give a result for $\lim_{\Lambda \rightarrow \infty} (\sigma_{FB,\Lambda} - \sigma_{FB,\Lambda}^{(1),nr})$ lying only 2% below the final answer. The results of the purely nonrelativistic calculations are compared to our approach in Fig. 3.8 for $m_t = 180$ and an electron polarization of 0.6. We also included the hard corrections as given in [6, 57].

We believe that our approach has the decisive advantage that it can be based upon to the *original* set of graphs to be considered and it is clear which graph has been calculated to which accuracy. Thus at least in principle a systematic improvement is possible. The present approach should also be applicable to the $O(\alpha_s)$ final state corrections calculated in [52].

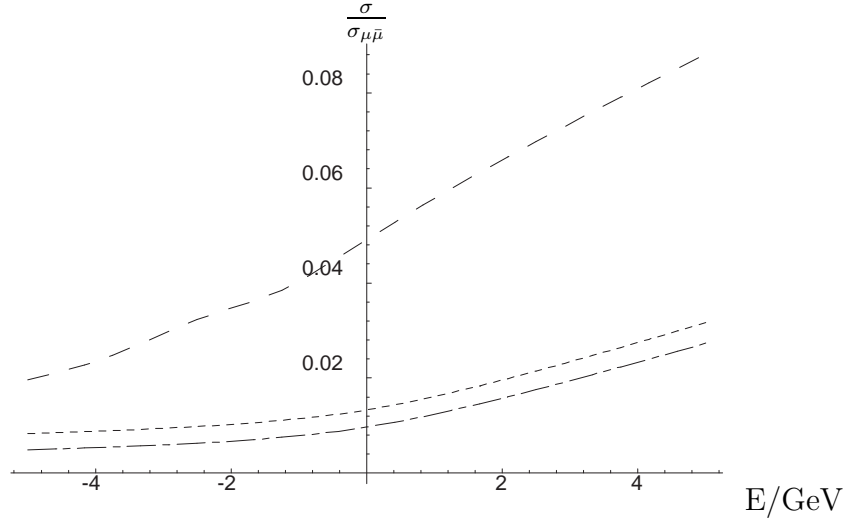


Fig. 3.7: Different Contributions to σ_{FB}
dashed: σ_{FB}^∞ , dotted: $\sigma_{FB}^{(1),nr}$, dashed-dotted: $\sigma_{FB}^{(1)}$

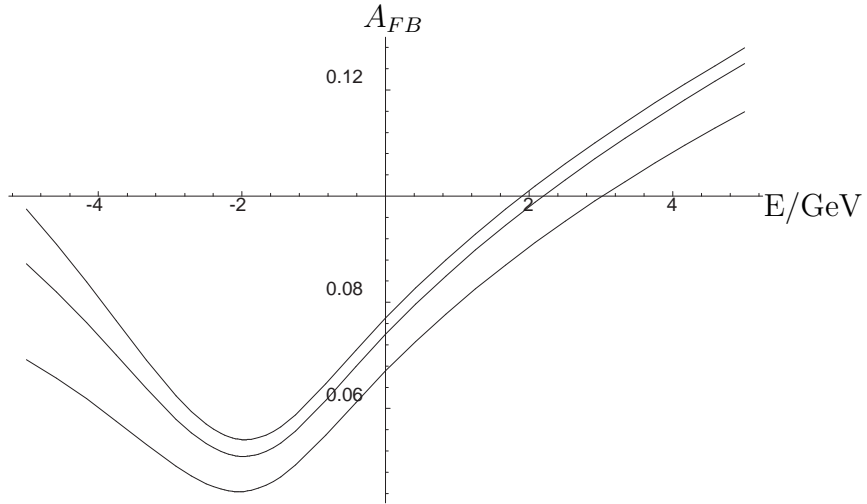


Fig. 3.8: Forward-backward asymmetry A_{FB} ;
curves from top to bottom: present work, ref.[56], ref.[57]

3.2 Axial Contribution to the $t\bar{t}$ Total Cross Section and the Production of Stop-Antistop near Threshold

The total cross section for the production of P-wave states near threshold gives the leading term for the production of stop-antistop and a contribution of $O(\alpha_s^2)$ to the total cross section for $t\bar{t}$. Only a qualitative investigation on the basis of the Coulomb Green function for the former has been published up to now [61]. We will apply the method of replacing divergent nonrelativistic graphs, implicit in the Green function approach, by the relativistic, finite ones to get a quantitative reliable result.

Considering the Coulomb green function as a first approximation it was observed in [61] that the total cross section for a stop-antistop pair near threshold develops an unphysical linear divergence due to the nonrelativistic approximation. Even a relativistic calculation of the imaginary part of the one loop contribution using the commonly used propagator $1/(p^2 - m^2 + im\Gamma)$ remains logarithmically divergent. This is due to the fact that this propagator does not fulfill the requirements of quantum field theory as does the propagator

$$\int \frac{d\mu^2}{\pi} \frac{m\Gamma}{(\mu^2 - m^2)^2 + m^2\Gamma^2} \frac{1}{p^2 - \mu^2 + i\epsilon}, \quad (3.82)$$

which is in agreement with the Lehmann representation of the full propagator. The tree contribution using this propagator reads

$$\sigma_1 = \frac{3}{2}(c_s + d_s\rho) \int d\mu_1^2 \int d\mu_2^2 \Delta(\mu_1^2) \Delta(\mu_2^2) \Theta(P^2 - (\mu_1 - \mu_2)^2) \left[1 - 2\frac{\mu_1^2 + \mu_2^2}{P^2} + \frac{(\mu_1^2 - \mu_2^2)^2}{P^4} \right]^{\frac{3}{2}} \sigma_{\mu\bar{\mu}}, \quad (3.83)$$

whereas we have for the nonrelativistic tree contribution

$$\sigma_{1,nr}^\Lambda = \frac{3\Gamma}{\pi m^2} (c_s + d_s\rho) \int_0^\Lambda dp \frac{p^4}{(p^2 - mE)^2 + m^2\Gamma^2} \sigma_{\mu\bar{\mu}}. \quad (3.84)$$

However, a single subtraction of the tree graph is not sufficient. One should also subtract the logarithmically divergent one-gluon exchange term. But since we replaced the gluon propagator by the resummed one we should also calculate the relativistic graph with a resummed propagator to get the correct behavior at infinity. Unfortunately this leads to renormalon ambiguities. Therefore, we conclude that the uncertainties in choosing the right cut-off are of the same order of magnitude as higher QCD corrections, and we

circumvent these difficulties for the time being by keeping the cut-off in the logarithmic divergent terms, choosing its value $\Lambda = \Lambda_0 = m$ from the observation that this cut-off would have given the correct answer in the case of the FB-asymmetry discussed in the last section.

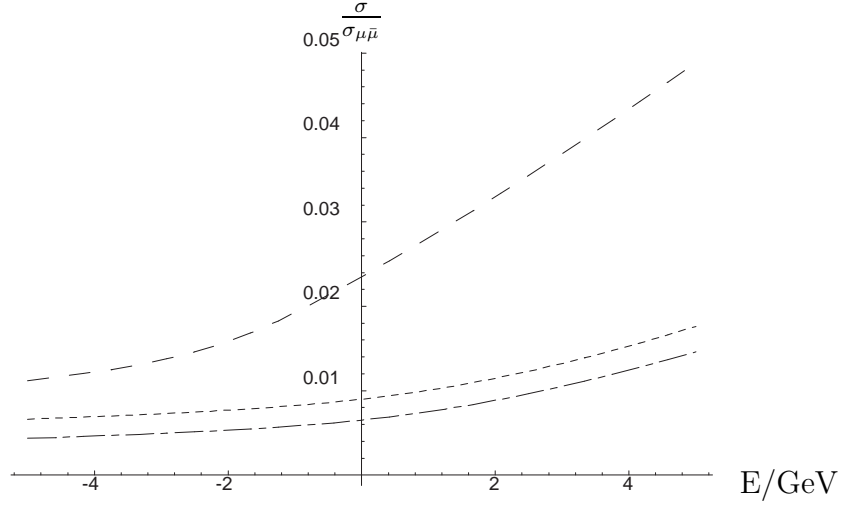


Fig. 3.9: Different Contributions to $\sigma_{t\bar{t}}$
dashed: $\sigma_{nr}^{\Lambda_0}$, dotted: $\sigma_{1,nr}^{\Lambda_0}$, dashed-dotted: $\sigma_{1,rel}$

The same is true for the pure axial contribution to the total cross section for $t\bar{t}$ production since each axial vertex contributes in the nonrelativistic limit an extra power in \vec{p} as explained in the last section. The only change is to replace the factor

$$f_s := \frac{3}{2}(c_s + d_s\rho) \quad (3.85)$$

by

$$f_f := 6(c_A + d_A\rho). \quad (3.86)$$

Using the methods of the preceding sections we get within the Green function approach

$$\sigma_{nr}^{\Lambda} = f_X \frac{3\Gamma}{2\pi^2 m^2} \int_0^{\Lambda} dp p^2 \int d\Omega \left| \int \frac{d^3 q}{(2\pi)^3} \frac{q^{(3)}}{m} G(\vec{q}, \vec{p}) \right|^2 \sigma_{\mu\bar{\mu}}, \quad (3.87)$$

where f_X denotes either f_s or f_f . The different contributions to the cross section for the scalar case

$$\sigma = \sigma_{nr}^{\Lambda_0} - \sigma_{1,nr}^{\Lambda_0} + \sigma_1 \quad (3.88)$$

are depicted in fig. 3.9 for $\cos^2 \theta_t = 0.5$, $m = 180$, $\Gamma = \Gamma_{top}$. The net result is shown in fig 3.10 for the axial contribution to the $t\bar{t}$ cross section.

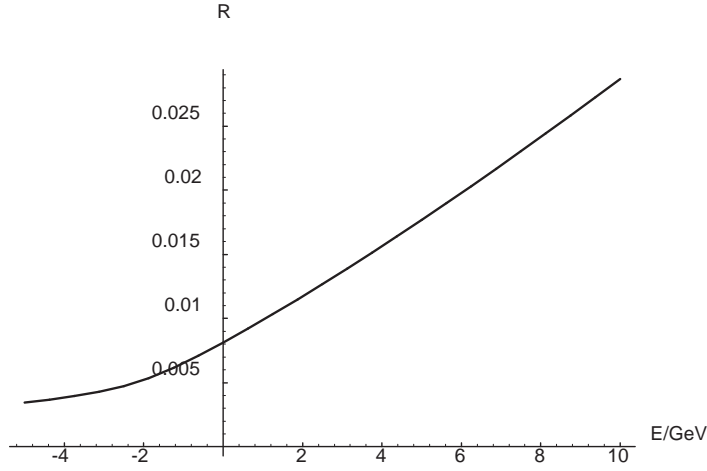


Fig. 3.10: Drell-ratio ($R = \sigma_A / \sigma_{\mu\bar{\mu}}$) for the production of $t\bar{t}$

According to ref. [62] we included the hard corrections by a factor $(1 - \frac{\alpha}{\pi})^2$. We conclude that the axial contribution gives a sizeable effect (of a few percent) to σ_{tot} beginning at $\approx 5\text{GeV}$ above threshold. However, it is small enough in order to hide the uncertainty in Λ with respect to $O(\alpha_s^2)$ corrections to the vector contribution to σ_{tot} .

Finally in fig. 3.11 we compare the cross sections for the production of $t\bar{t}$ near threshold for two different values of the decay constant ($\cos^2 \theta_t = 0.5, m = 180$).

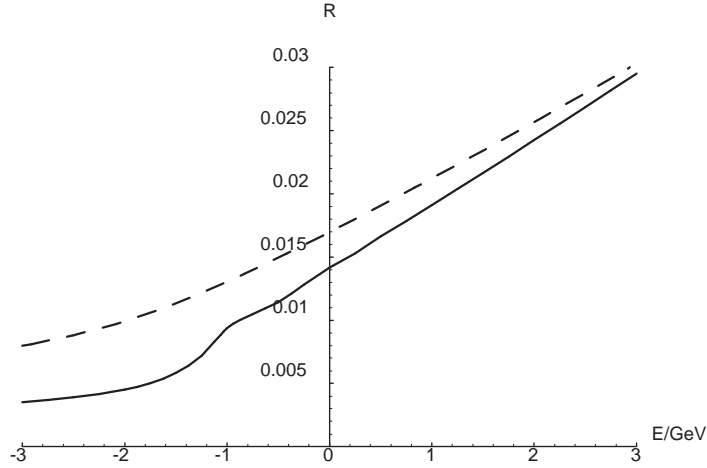


Fig. 3.11: dashed curve: $\Gamma = 1.5\text{GeV}$, full line: $\Gamma = 0.5\text{GeV}$

Chapter 4

Conclusion

In this thesis we considered the quantum field theory near thresholds of ultra-heavy particles, where we especially focused on the $t\bar{t}$ and $t\bar{t}$ systems. Due to the large mass and the large width of the top quark it is possible to treat the former in a pure perturbative manner although the top is a strongly interacting particle. The large mass of the top close to the weak scale may be an indicator of new physics. Therefore, it is of paramount interest to predict physical observables within the framework of the existing Standard Model in order to be able to single out possible new effects.

In this context the question of the possible large corrections due to a running width of the top quark could be solved in this work. It was shown that a previously not considered contribution leads to large cancellations and leads especially to a gauge independent result. This was first achieved in the narrow width approximation and subsequently improved by the introduction of a new relativistic zero order equation already including the on-shell decay width. With the help of that equation and a Ward identity it was further possible to show that the bound state corrections to the decay width for a generic fermionic system which does not decay by annihilation is given by

$$\Delta\Gamma = -\Gamma_0 \frac{\langle \vec{p}^2 \rangle}{2m^2}. \quad (4.1)$$

Furthermore a systematic derivation of the $t\bar{t}$ potential to numerical order $O(\alpha_s^4)$ was given. Especially the contribution of a QCD box graph and a thorough investigation of effects of the weak interaction are new. Also the potential for scalar-scalar bound states has been investigated with the help of a new zero order equation for such systems. A deviation from a previous result was found.

In the second part of this work we considered within the numerical Green function approach the cross sections for the production of ultraheavy particles near threshold. First we derive the general formalism and consider the total cross section. Here the large width is demonstrated explicitly to be very important for hiding nonperturbative effects. Then we investigate the forward-backward asymmetry near threshold. We avoid the introduction of an unnatural cut-off as contained in previous work. This is achieved by identifying the divergent graphs in the nonrelativistic Green function and replacing them by the relativistic ones. The same method can also be applied to the pure axial contribution to the $t\bar{t}$ total cross section as well as to the stop-antistop production. This enables us to give first quantitative predictions of these observables.

While some aspects of the production of heavy particles near threshold are well understood by now, there remain several unsolved problems. Especially proceeding to higher orders in perturbation theory seems rather difficult in view of the necessity of including the leading Coulomb interaction to all orders. It has not been possible to completely combine the rigorous bound state approach of Chapter 2 with the practically tractable numerical approach of Chapter 3 in a satisfactory way, yet. Nevertheless, we hope that this work provides a basis for further investigations in this direction.

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Appendix A

Expectation Values

In sect. 2.4 and 3.1 we needed the expectation values of logarithmic potentials between Schrödinger wave functions. They can be obtained by

$$\langle \frac{\ln^n r}{r} \rangle = \frac{d^n}{d\lambda^n} \langle r^{\lambda-1} \rangle|_{\lambda=0}, \quad (\text{A.1})$$

if the expectation value $\langle r^{\lambda-1} \rangle$ is known analytically. In the latter the representation

$$L_{n-l-1}^{2l+1}(\rho) = \lim_{z \rightarrow 0} \frac{1}{(n-l-1)!} \frac{d^{n-l-1}}{dz^{n-l-1}} (1-z)^{-2l-2} e^{\rho \frac{z}{z-1}} \quad (\text{A.2})$$

of the Laguerre polynomials may be used. This allows an easy evaluation of the integrations and the remaining differentiations can be done with some care afterwards:

$$\langle r^{\lambda-1} \rangle = \frac{(\alpha m)^{1-\lambda}}{2n^{2-\lambda}} \frac{(n-l-1)!}{(n+l)!} \Gamma(2l+2+\lambda) \sum_{k=0}^{n-l-1} \binom{\lambda}{n-l-1-k}^2 \binom{-2l-2-\lambda}{k} (-1)^k \quad (\text{A.3})$$

Using now the Fourier transformations [63]

$$F\left[\frac{\ln \frac{\vec{q}^2}{\mu^2}}{\vec{q}^2}\right] = -\frac{\gamma + \ln \mu r}{2\pi r} \quad (\text{A.4})$$

$$F\left[\frac{\ln^2 \frac{\vec{q}^2}{\mu^2}}{\vec{q}^2}\right] = \frac{1}{2\pi r} \left[\frac{\pi^2}{6} + 2(\gamma + \ln \mu r)^2 \right] \quad (\text{A.5})$$

one arrives immediately at eq.3.46 and 2.83, respectively.

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